Reinforcement Learning
From the basics to Deep RL

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Outline

- Some quick background about discrete RL and actor-critic methods
- DQN and the main tricks
- Beyond DQN: a few state-of-the-art papers
- What is DDPG, how does it work?
- Further algorithms: NAF, TRPO, ...
Supervised learning

- The supervisor indicates to the agent **the expected answer**
- The agent **corrects a model** based on the answer
- Typical mechanism: gradient backpropagation, RLS
- Applications: classification, regression, function approximation...
Cost-Sensitive Learning

- The environment provides the value of action (reward, penalty)
- Application: behaviour optimization
In RL, the value signal is given as a scalar

How good is -10.45?

Necessity of exploration
The exploration/exploitation trade-off

- Exploring can be (very) harmful
- Shall I exploit what I know or look for a better policy?
- Am I optimal? Shall I keep exploring or stop?
- Decrease the rate of exploration along time
- $\epsilon$-greedy: take the best action most of the time, and a random action from time to time
Markov Decision Processes

- $S$: states space
- $A$: action space
- $T : S \times A \rightarrow \Pi(S)$: transition function
- $r : S \times A \rightarrow \mathbb{R}$: reward function

- An MDP defines $s^{t+1}$ and $r^{t+1}$ as $f(s_t, a_t)$
- It describes a problem, not a solution
- Markov property: $p(s^{t+1}|s^t, a^t) = p(s^{t+1}|s^t, a^t, s^{t-1}, a^{t-1}, ... s^0, a^0)$
- Reactive agents $a_{t+1} = f(s_t)$, without internal states nor memory
- In an MDP, a memory of the past does not provide any useful advantage
Policy and value functions

- **Goal:** find a policy \( \pi : S \rightarrow A \) maximizing the aggregation of reward on the long run.

- **The value function** \( V^\pi : S \rightarrow \mathbb{R} \) records the aggregation of reward on the long run for each state (following policy \( \pi \)). It is a vector with one entry per state.

- **The action value function** \( Q^\pi : S \times A \rightarrow \mathbb{R} \) records the aggregation of reward on the long run for doing each action in each state (and then following policy \( \pi \)). It is a matrix with one entry per state and per action.
Reinforcement learning

- In Dynamic Programming (planning), $T$ and $r$ are given
- Reinforcement learning goal: build $\pi^*$ without knowing $T$ and $r$
- Model-free approach: build $\pi^*$ without estimating $T$ nor $r$
- Actor-critic approach: special case of model-free
- Model-based approach: build a model of $T$ and $r$ and use it to improve the policy
Families of methods

- **Critic**: (action) value function $\rightarrow$ evaluation of the policy
- **Actor**: the policy itself
- **Critic-only methods**: iterates on the value function up to convergence without storing policy, then computes optimal policy. Typical examples: value iteration, Q-learning, Sarsa
- **Actor-only methods**: explore the space of policy parameters. Typical example: CMA-ES
- **Actor-critic methods**: update in parallel one structure for the actor and one for the critic. Typical examples: policy iteration, many AC algorithms
- Q-learning and Sarsa look for a global optimum, AC looks for a local one
Incremental estimation

- Estimating the average immediate (stochastic) reward in a state $s$
  
  $E_k(s) = (r_1 + r_2 + \ldots + r_k)/k$

  $E_{k+1}(s) = (r_1 + r_2 + \ldots + r_k + r_{k+1})/(k + 1)$

  Thus $E_{k+1}(s) = k/(k + 1)E_k(s) + r_{k+1}/(k + 1)$

  Or $E_{k+1}(s) = (k + 1)/(k + 1)E_k(s) - E_k(s)/(k + 1) + r_{k+1}/(k + 1)$

  Or $E_{k+1}(s) = E_k(s) + 1/(k + 1)[r_{k+1} - E_k(s)]$

- Still needs to store $k$

- Can be approximated as

  $$E_{k+1}(s) = E_k(s) + \alpha [r_{k+1} - E_k(s)]$$  \hspace{1cm} (1)

  Converges to the true average (slower or faster depending on $\alpha$) without storing anything

- Equation (1) is everywhere in reinforcement learning
Temporal Difference Error

- The goal of TD methods is to estimate the value function $V(s)$
- If estimations $V(s_t)$ and $V(s_{t+1})$ were exact, we would get:
  - $V(s_t) = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} + ...$
  - $V(s_{t+1}) = r_{t+2} + \gamma(r_{t+3} + \gamma^2 r_{t+4} + ...$
  - Thus $V(s_t) = r_{t+1} + \gamma V(s_{t+1})$
- $\delta_k = r_{k+1} + \gamma V(s_{k+1}) - V(s_k)$: Reward Prediction Error (RPE)
- Measures the error between current and expected values of $V$
- TD learning: If $\delta$ positive, increase $V$, if negative, decrease $V$
- $V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)]$
TD learning: limitation

- TD(0) evaluates $V(s)$
- One cannot infer $\pi(s)$ from $V(s)$ without knowing $T$: one must know which $a$ leads to the best $V(s')$
- Three solutions:
  - Work with $Q(s, a)$ rather than $V(s)$ (Sarsa and Q-Learning)
  - Learn a model of $T$: model-based (or indirect) reinforcement learning
  - Actor-critic methods (simultaneously learn $V$ and update $\pi$)
Sarsa

- Reminder (TD): \( V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \)
- Sarsa: For each observed \((s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})\):
  \[ Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \]
- Policy: perform exploration (e.g. \( \epsilon\)-greedy)
- One must know the action \( a_{t+1} \), thus constrains exploration
- On-policy method: more complex convergence proof

Reinforcement Learning

Background

General RL background

Q-Learning

- For each observed \((s_t, a_t, r_{t+1}, s_{t+1})\):

\[
Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[ r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t) \right]
\]

- \(\max_{a \in A} Q(s_{t+1}, a)\) instead of \(Q(s_{t+1}, a_{t+1})\)
- Off-policy method: no more need to know \(a_{t+1}\)
- Policy: perform exploration (e.g. \(\epsilon\)-greedy)
- Convergence proved given infinite exploration [Dayan & Sejnowski, 1994]

**Q-Learning in practice**

(Q-learning: the movie)

- Build a states × actions table (**$Q$-Table**, eventually incremental)
- Initialise it (randomly or with 0 is not a good choice)
- Apply update equation after each action
- Problem: it is (very) slow
Model-based Reinforcement Learning

- General idea: planning with a learnt model of $T$ and $r$ is performing back-ups “in the agent’s head” ([Sutton, 1990, Sutton, 1991])
- Learning $T$ and $r$ is an incremental self-supervised learning problem
- Several approaches:
  - Draw random transition in the model and apply TD back-ups
  - Dyna-PI, Dyna-Q, Dyna-AC
  - Better propagation: Prioritized Sweeping

Dyna architecture and generalization

(Dyna-like video (good model))
(Dyna-like video (bad model))

- Thanks to the model of transitions, Dyna can propagate values more often
- Problem: in the stochastic case, the model of transitions is in \( \text{card}(S) \times \text{card}(S) \times \text{card}(A) \)
- Usefulness of compact models
- MACS: Dyna with generalisation (Learning Classifier Systems)
- SPITI: Dyna with generalisation (Factored MDPs)


From *Q-Learning* to *Actor-Critic* (1)

<table>
<thead>
<tr>
<th>state / action</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_0$</td>
<td>0.66</td>
<td>0.88</td>
<td>0.81</td>
<td>0.73</td>
</tr>
<tr>
<td>$e_1$</td>
<td>0.73</td>
<td>0.63</td>
<td>0.9</td>
<td>0.43</td>
</tr>
<tr>
<td>$e_2$</td>
<td>0.73</td>
<td>0.9</td>
<td>0.95</td>
<td>0.73</td>
</tr>
<tr>
<td>$e_3$</td>
<td>0.81</td>
<td>0.9</td>
<td>1.0</td>
<td>0.81</td>
</tr>
<tr>
<td>$e_4$</td>
<td>0.81</td>
<td>1.0</td>
<td>0.81</td>
<td>0.9</td>
</tr>
<tr>
<td>$e_5$</td>
<td>0.9</td>
<td>1.0</td>
<td>0.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>

- In *Q-learning*, given a *Q-Table*, one must determine the max at each step.
- This becomes expensive if there are numerous actions (optimization in continuous action case).
From *Q-Learning* to Actor-Critic (2)

<table>
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<td>0.9</td>
</tr>
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</table>

- One can store the best value for each state
- Storing the max is equivalent to storing the policy
- Update the policy as a function of value updates (only look for the max when decreasing max action)
- Note: looks for local optima, not global ones anymore
Naive actor-critic approach

- Discrete states and actions, stochastic policy
- An update in the critic generates a local update in the actor
- Critic: compute $\delta$ and update $V(s)$ with $V_k(s) \leftarrow V_k(s) + \alpha_k \delta_k$
- Actor: $P^\pi(a|s) = P^\pi(a|s) + \alpha_k \delta_k$
- NB: no need for a max over actions, but local maximum
- NB2: one must know how to “draw” an action from a probabilistic policy (not straightforward for continuous actions)
A few messages

- Dynamic programming and reinforcement learning methods can be split into pure actor, pure critic and actor-critic methods
- Dynamic programming, value iteration, policy iteration are when you know the transition and reward functions
- Actor critic RL is a model-free, PI-like algorithm
- Model-based RL combines dynamic programming and model learning
Questions

- SARSA is on-policy and Q-learning is off-policy
  Right or Wrong?

- The actor-critic approach is model-based
  Right or Wrong?

- In SARSA, the policy is represented implicitly through the critic
  Right or Wrong?
Parametrized representations

- To represent a continuous function, use features and a vector of weights (parameters)
- Learning tunes the weights

- Linear architecture: linear combination of features
- A deep neural network is not a linear architecture: weights also “inside” the features
- Two parametrized representations:
  - In policy gradient methods: of the policy $\pi_w(a_t|s_t)$
  - In actor-critic methods: and also of the critic $Q(s_t, a_t|\theta)$
Optimization over continuous actions

- In RL, you need a max over actions
- If the action space is continuous, this is a difficult optimization problem
- Policy gradient methods and actor-critic methods mitigate the problem by looking for a local optimum (Pontryagin methods vs Bellman methods)
Quick history of previous attempts (J. Peters’ and Sutton’s groups)

- Those methods proved inefficient for robot RL
- Keys issues: value function estimation based on linear regression is too inaccurate, tuning the stepsize is critical

General motivations for Deep RL

- Approximation with deep networks provided enough computational power can be very accurate
- Discover the adequate features of the state in a large observation space
- All the processes rely on efficient backpropagation in deep networks
- Available in CPU/GPU libraries: TensorFlow, theano, caffe, Torch... (RProp, RMSProp, Adagrad, Adam...)
DQN: the breakthrough

- DQN: Atari domain, Nature paper, small discrete actions set
- Learned very different representations with the same tuning

The Q-network in DQN

- Limitation: requires one output neuron per action
- Select action by finding the max (as in Q-Learning)
- Q-network parameterized by $\theta$
Learning the Q-function

- Supervised learning: minimize a loss-function, often the squared error w.r.t. the output:

\[ L(s, a) = (y^*(s, a) - Q(s, a|\theta))^2 \]  

(2)

by backprop on critic weights \( \theta \)

- For each sample \( i \), the Q-network should minimize the RPE:

\[ \delta_i = r_i + \gamma \max_a Q(s_{i+1}, a|\theta) - Q(s_i, a_i|\theta) \]

- Thus, given a minibatch of \( N \) samples \( \{s_i, a_i, r_i, s_{i+1}\} \), compute

\[ y_i = r_i + \gamma \max_a Q(s_{i+1}, a|\theta') \]

- So update \( \theta \) by minimizing the loss function

\[ L = 1/N \sum_i (y_i - Q(s_i, a_i|\theta))^2 \]  

(3)
Trick 1: Stable Target Q-function

- $y_i = r_i + \gamma \max_a Q(s_{i+1}, a)|\theta)$ is a function of $Q$
- Thus this is not truly supervised learning, and this is unstable
- Idea: compute the critic loss function from a separate target network $Q'(\ldots|\theta')$
- So rather compute $y_i = r_i + \gamma \max_a Q'(s_{i+1}, a)|\theta')$
- $\theta$ is updated only each $K$ iterations (so “periods of supervised learning”)

\[\]
In most optimization algorithms, samples are assumed independently and identically distributed (iid)

- Obviously, this is not the case of behavioral samples \((s_i, a_i, r_i, s_{i+1})\)
- Idea: put the samples into a buffer, and extract them randomly
- Use training minibatches, to make profit of GPU
- The replay buffer management policy is an issue
Double-DQN

- The max operator in the RPE results in the propagation of over-estimation
- This max operator is used both for action choice and value propagation
- Double \textit{Q-Learning}: separate both calculations (Van Hasselt)
- Double-DQN: make profit of the target network: propagate on target network, select max on Q-network,
- Minor change with respect to DQN
- But with a much better performance
- Recent paper on double SARSA


Prioritized Experience Replay

- Samples with a greater TD error have a higher probability of being selected
- Favors the replay of new \((s, a)\) pairs (largest TD error), as in \(R – \max\)
- Several minor hacks, and interesting discussion
- Converges twice faster


- Other state-of-the-art methods: Gorilla, A3C: parallel implementations without replay buffers

DDPG: The paper

- Continuous control with deep reinforcement learning
- Timothy P. Lillicrap, Jonathan J. Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, Daan Wierstra
- Google Deepmind
- On arXiv since september 7, 2015
- Already cited > 280 times

Applications: impressive results

- End-to-end policies (from pixels to control)
- Works impressively well on “More than 20” (27-32) such domains
- Some domains coded with MuJoCo (Todorov) / TORCS
- OpenAI gym gives access to those benchmarks

DDPG: ancestors

- Most of the actor-critic theory for continuous problem is for stochastic policies (policy gradient theorem, compatible features, etc.)
- DPG: an efficient gradient computation for deterministic policies, with proof of convergence

General architecture

- Any neural network structure
- Actor parametrized by $w$, critic by $\theta$
- All updates based on backprop
Training the critic

- Same idea as in DQN, but with an actor-critic update rather than Q-Learning
- Minimize the RPE: $\delta_t = r_t + \gamma Q(s_{t+1}, \pi(s_t)|\theta) - Q(s_t, a_t|\theta)$
- Given a minibatch of $N$ samples $\{s_i, a_i, r_i, s_{i+1}\}$ and a target network $Q'$, compute $y_i = r_i + \gamma Q'(s_{i+1}, \pi(s_{i+1})|\theta')$
- And update $\theta$ by minimizing the loss function

$$L = \frac{1}{N} \sum_i (y_i - Q(s_i, a_i|\theta))^2$$
From DQN: Target network

- In DDPG, instead of scarce updates, slow evolution of $Q'$ and $\pi'$

$$\theta' \leftarrow \tau \theta + (1 - \tau) \theta'$$

- The same applies to $\mu$, $\mu'$ (slow evolution of the actor)
- From the empirical study, this is a critical trick
- NB: actor-critic tuning is known to be tedious!

Training the actor

Deterministic policy gradient theorem: the true policy gradient is
\[ \nabla_w \pi(s, a) = \mathbb{E}_{\rho(s)}[\nabla_a Q(s, a|\theta) \nabla_w \pi(s|w)] \] (5)

- \( \nabla_a Q(s, a|\theta) \) is obtained by computing the gradient over actions of \( Q(s, a|\theta) \)
- Gradient over actions \( \sim \) gradient over weights (symmetric roles of weights and inputs)
- \( \nabla_a Q(s, a|\theta) \) is used as backprop error signal to update the actor weights.
- Comes from NFQCA

General algorithm

1. Feed the actor with the state, outputs the action
2. Feed the critic with the state and action, determines $Q(s, a|\theta^Q)$
3. Update the critic, using (4) (alternative: do it after 4?)
4. Compute $\nabla_a Q(s, a|\theta)$
5. Update the actor, using (5)
Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights $\theta^Q$ and $\theta^\mu$.
Initialize target network $Q'$ and $\mu'$ with weights $\theta^Q \leftarrow \theta^Q$, $\theta^\mu \leftarrow \theta^\mu$
Initialize replay buffer $R$

for episode = 1, M do
    Initialize a random process $\mathcal{N}$ for action exploration
    Receive initial observation state $s_1$

    for $t = 1, T$ do
        Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
        Execute action $a_t$ and observe reward $r_t$ and observe new state $s_{t+1}$
        Store transition $(s_t, a_t, r_t, s_{t+1})$ in $R$
        Sample a random minibatch of $N$ transitions $(s_i, a_i, r_i, s_{i+1})$ from $R$
        Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^\mu')|\theta^Q')$
        Update critic by minimizing the loss: $L = \frac{1}{N} \sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
        Update the actor policy using the sampled gradient:
        $$\nabla_{\theta^\mu} \mu |_{s_i} \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu (s|\theta^\mu)|_{s_i}\n$$

        Update the target networks:
        $$\theta^Q' \leftarrow \tau \theta^Q + (1 - \tau) \theta^Q'$$
        $$\theta^\mu' \leftarrow \tau \theta^\mu + (1 - \tau) \theta^\mu'$$

end for
end for

Notice the slow $\theta'$ and $\mu'$ updates (instead of copying as in DQN)
Subtleties

- The actor update rule is
  \[ \nabla_w \pi(s_i) \approx 1/N \sum_i \nabla_a Q(s, a|\theta)|_{s=s_i, a=\pi(s_i)} \nabla_w \pi(s)|_{s=s_i} \]

- Thus we do not use the action in the samples to update the actor.

- Could it be
  \[ \nabla_w \pi(s_i) \approx 1/N \sum_i \nabla_a Q(s, a|\theta)|_{s=s_i, a=a_i} \nabla_w \pi(s)|_{s=s_i} ? \]

- Work on \( \pi(s_i) \) instead of \( a_i \).

- Does this make the algorithm on-policy instead of off-policy?

- Does this make a difference?
Trick 3: Batch Normalization

- Covariate shift: as layer $N$ is trained, the input distribution of layer $N + 1$ is shifted, which makes learning harder
- To fight covariate shift, ensure that each dimension across the samples in a minibatch have unit mean and variance at each layer
- Add a buffer between each layer, and normalize all samples in these buffers
- Makes learning easier and faster
- Makes the algorithm more domain-insensitive
- But poor theoretical grounding, and makes network computation slower

Back to natural gradient: other ideas

- Using the advantage function leads to natural gradient (vs vanilla gradient)
- Batch normalization and Weight normalization are specific reparametrization methods
- Computing the natural gradient is also a reparametrization method
- Natural Neural networks define a reparametrization that compute the natural gradient (to be investigated)


NAF: Approximate the advantage function

- Reminder: in *Q-Learning*, high cost to select best action
- Here, set a specific form to Q-network so as to find the best action easily
- Advantage function: \( A(s_i, a_i|\theta) = Q(s_i, a_i|\theta) - \max_a Q(s_i, a|\theta) \)
- \( V(s_i) = \max_a Q(s_i, a|\theta) \)
- \( Q(s_i, a_i|\theta^Q) = A(s_i, a_i|\theta^A) + V(s_i|\theta^V) \)
- \( A_\theta(s_i, a_i|\theta^A) = \frac{1}{2} (a_i - \mu(s_i|\theta^\mu))^T P(s_i|\theta^P)(a_i - \mu(s_i|\theta^\mu)) \)

NAF: the network

- All neural nets are $\text{dim}(s) \times \text{dim}(a)$
- Implemented with 2 layers of 200 relu units
- The $\mu$ network is the actor
- Outperforms DDPG on some benchmarks
- Other tricks in the paper: use iLQG for model-based acceleration
Status

- DDPG used successfully on continuous Mountain Car: much more data efficient than CMA-ES
- I failed to tune it for a 4D/6D motor control problem with noisy perception and delays
- NAF is used in real robotics settings with some success
- Now working accurately on the stability issue

TRPO, PPO

- Theory: monotonous improvement w.r.t. the cost function
- Practice: good grip on the step size
- Follows the natural gradient
- More stable, performs well in practice


The frontiers

- Two more recent papers: ACER and Q-Prop
- Confirm that DDPG is tricky to tune
- Combine the TRPO and DDPG approaches to get more efficient and more stable
- It gets really complicated
- The fundamental instability issue is not solved
- One cannot compete with OpenAI, Google US and Google DeepMind on this topic...


Reinforcement learning for robots (old)
Reinforcement learning for robots (new)
Any question?
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