Modèles de l'apprentissage et du contrôle sensori-moteur

Computational motor control

2nd course

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Dynamical systems

- Internal state. E.g: point mass subject to a force. To define the movement of the point, its mass, the history of applied forces, and initial conditions (position, velocity) must be known.
 - Position and velocity are the states of the system.
 - Why state? The human body includes ~600 muscles which are contracted or not. There exists 2^600 motor activations.
 - Internal state provides a simplified and compact representation of the state of the system.

x[n]	state	y[n]	output (observation)
u[n]	input (co	ontrol)	
x[n+1] = f(x[n], u[n])			state equation
y[n] = g(x[n])			output equation
y[n +	1] = h(x[n], u[n])	

Dynamical systems (end)

- **Control**. Find an input (control) in order to obtain a given output (observation) behavior.
 - Stay close to a reference value: regulator; stay close to a trajectory: tracking problem.
- Estimation. Input can be calculated from the state if available.
 - In general, the state is unknown and can only be estimated. Estimation builds a representation of the current state from the history of outputs (observations).
- Feedback. An input has an expected effect that may not occur due to perturbations or inexact knowledge of the system.
 - A feedback is necessary to compare the actual and desired performance, and produce a compensatory input. Feedback should be appropriate (instability, poor behavior).

Closed loop control

• Well-defined error, limit on the gain, delay in the loop, no prediction. Used for slow movements, posture.

$$u_{fb}[n] = K(y^*[n] - y[n])$$



- Types of control
 - Proportionnal $au(t) = K \left[\theta_d(t) \theta(t) \right]$
 - Derivative: damping to reduce oscillations

$$\tau(t) = K_p \left[\theta_d(t) - \theta(t)\right] - K_v \dot{\theta}(t)$$

- Integral: to avoid steady state error

$$\tau(t) = K_p \left[\theta_d(t) - \theta(t)\right] - K_v \dot{\theta}(t) + K_i \int_{t_o}^t \left[\theta_d(t) - \theta(t)\right] dt$$

Closed loop control (...)



Open loop control

- E.g solution of the inverse dynamics
 - Problem when (1) the model of the dynamics is not exact; (2) initial conditions are different from what has been planned; (3) unmodeled/unexpected perturbations are present. Complex calculus, no generalization. Used for fast movements.

$$u_{ff}[n] = \phi_{ff}(\hat{x}[n], y^*[n+1]) \qquad \phi_{ff} = \hat{h}^{-1}$$
$$e[n+1] = y^*[n+1] - h\left(x[n], \hat{h}^{-1}(\hat{x}[n], y^*[n+1])\right)$$



Open loop control (...)



Comparison

- Characteristics
 - Predictive control | error correction
 - Model-based | no model
 - Sensitive to modeling uncertainty | not sensitive
 - Sensitive to unexpected/unmodeled perturbations | robust
- Hybrid control



Causality

- Choice of input and output variables. Relies on the nature of causality.
 - E.g., in a given model, a muscular activation is an input and the corresponding joint torque is an output.
 - In a different model, joint torque is an input, and the corresponding displacement is an output.
- Causality can be extended to functional relationships between variables.
 - Direct kinematics (joint coordinates \rightarrow spatial coordinates). Direct transformation.
 - In the redundant case, inverse kinematics is not a function. Yet it interesting and useful to use the notion of inverse kinematics.

Internal models

- **Direct model**: model of the causal relationship between actions and their consequences. Useful to predict the behavior of a system (body, world, ...).
 - A direct model of arm dynamics combines the current state of the arm (position, velocity), and the current input (control) to predict the future state of the arm (position, velocity).
- **Inverse Model**: model of the relationship between desired consequences and corresponding actions.
 - An inverse model of the arm dynamics translates a desired trajectory into appropriate inputs (controls) to drive the arm along this trajectory.

Internal models (...)



[state, motor command, context] --- sensory feedback

Wolpert & Ghahramani (2000)

Existence of direct models

To prevent a manipulated object to slip during movement, a **grip force** must be exerted to compensate for the **load force**.



Wolpert & Flanagan (2001)

Existence of direct models

A subject creates a tactile stimulation on one hand through a robotic device actuated by the other hand. When the transmission is direct, the subject can subtract the predicted sensory effect from the actual sensory effect due to the tactile stimulation. The subject perceives no tickling.

When a delay is added by the device, the subject perceives a prediction error that is interpretated as a tickling sensation.



Wolpert & Flanagan (2001)

Existence of direct models

Subjects estimate the position of their hand at the end of a movement (no visual feedback). In some cases, a force is applied to the hand. Errors vary as function of movement duration, and are described by the time course of bias and variance. Combined use of a **direct model** (prediction of displacement based on motor commands) and sensory feedback. The data cannot be explained by one or the other component alone.







Motor

Role of direct models

• A system can use a direct model rather than an external feedfback to evaluate the effect of command and its associated error. Avoid the instability due to delays in feedback loops.

• Kalman filter.



Existence of inverse models

When a subject encounters a dynamical perturbation for the 1st time (e.g. force field), its movements are modified. With training, the movements progressively return to their normal shape.

When the perturbation is removed, the movements are again modified (aftereffects). These after-effects indicate that an inverse model of the system dynamics has been modified with training.



Shadmehr & Mussa-Ivaldi (1994)

Existence of inverse models

EMG activity is observed in muscles acting at non-moving joints during shoulder or elbow movements. This activity is agonist/antagonist similar to activity observed during movement. It starts before the movement and varies with the velocity of the moving segment (i.e. with the interaction torque by the moving produced segment).



Gribble & Ostry (1999)

Optimality principle

- The interaction between the behavior and the environment leads a better adaptation of the former to the latter. The tendency could lead to an optimal behavior, i.e. the best behavior corresponding to a goal, according to a given criterion.
- The idea is to describe a movement not in terms of its characteristics (kinematics, dynamics), but in an abstract way, using a global value to be maximized or minimized.
- E.g. smoothness, energy, variability, ...



Harris & Wolpert (1998)

Optimal control

Method to find the solution of an optimal control problem (minimum of a cost function).

E.g. Find the trajectory of maximum smoothness between two points. The optimal trajectory is straight with a bell-shaped velocity profile.

How does the nervous system calculate an optimal command? What is the cost function?

$$\begin{split} C(x(t)) &= \int_0^{t_f} L\left[t, x, \dot{x}, \ddot{x}, ..., \frac{d^n}{dt^n}\right] dt \\ &\frac{\partial L}{\partial x} - \frac{d}{dt} \left(\frac{\partial L}{\partial x}\right) ... + (-1)^n \frac{d^n}{dt^n} \left(\frac{\partial L}{\partial x^{(n)}}\right) = 0 \end{split}$$

Maximum of smoothness = minimum jerk

$$\begin{split} C &= \frac{1}{2} \int_0^{t_f} \left(\left[\frac{d^3 x}{dt^3} \right]^2 + \left[\frac{d^3 y}{dt^3} \right]^2 \right) dt \\ L &= \frac{1}{2} \left((\ddot{x})^2 + (\ddot{y})^2 \right) \\ \frac{d^3}{dt^3} \left(\frac{\partial \ddot{x}^2}{\partial \ddot{x}} \right) + \frac{d^3}{dt^3} \left(\frac{\partial \ddot{y}^2}{\partial \ddot{y}} \right) = 0 \\ \frac{d^6 x}{dt^6} &= \frac{d^6 y}{dt^6} = 0 \\ x(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \\ y(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{split}$$

Learning a forward model

A forward model uses a copy of the command to predict the consequences of an action. This prediction can be compared to the true consequence to generate an error signal. The error signal can be used to update the model.



Wolpert & Flanagan (2001)

Direct inverse learning

A transformation is learning by sampling the inverse transformation. E.g. learning the relationship *desired behavior* \rightarrow *command* from samples of the relationship *command* \rightarrow *behavior*. Learning and control phases are distinct.



Direct inverse learning (...)

Difficulties

- The learned model does not guarantee the correct execution of a desired behavior. In fact, all the desired behaviors may not have been encountered during the sampling *command* \rightarrow *behavior*.

- The model can fail to learn correctly the inverse model of a redundant system. If the set of commands associated to a given behavior is not convex, the command, obtained as the mean of admissible commands, may not be admissible.



Jordan & Rumelhart (1992)

Distal supervised learning

Actions are proximal variables (directly controlled by the student). The consequences of action are distal variables.

The student creates a direct model of the environment by exploring the results associated with different actions. This model is used to learn the relationship between intentions and actions.



Distal supervised learning (...)

Learning of the inverse model is based on performance error (difference between the expected and actual output). The direct model translates this error in the distal space into an error in the proximal space. The proximal error can be used to supervise learning of inverse model.



Distal supervised learning (end)

The structure of the model is a multilayered neural network. Learning is performed using gradient backpropagation.

The model can learn the inverse kinematics of a redundant system.



Jordan & Rumelhart (1992)

Feedback-error learning

A feedback controller is used to reduce the error between the current and desired state. A feedback command is added to the feedforward command generated by the inverse model.

The feedback command becomes null when there is no more error. It can be used as an error signal to train the inverse model.



Synthesis



Jordan & Wolpert (1999)