Modèles de l'apprentissage et du contrôle sensori-moteur

Models of motor control

4th course

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Levels

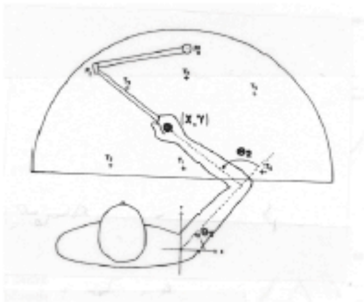
- Levels of Marr
  - Computational: abstract level of analysis in which a task can be shared into subtasks
  - Algorithmic: formal way to solve the task
  - Implementation: how the solution can be physically realized
- Mechanics
  - Body movements follow the laws of mechanics. Thus the knowledge of mechanics is necessary to study the neural bases of movements. Yet, a direct identification of equation terms to nervous processes is likely to be meaningless.
- Biomechanics
  - Knowledge of degrees of freedom, muscle characteristics ...

Levels (...)

- Muscle
  - How to describe muscular function for motor control (spring, force generator, ...)? What is the appropriate level of description?
- Muscle + reflex
  - Basic circuit for motor control?
- Spinal cord (neuromuscular system)
  - How to extract a function from the complex arrangement of spinal circuits?
- Principles
  - What are the principles that guide the functioning of the motor system?
- Architecture
  - Anatomo-functional circuits for motor control.

Mechanics

$$
\begin{aligned}
\tau_1 &= (I_1 + I_2 + m_2 l_1 l_2 \cos\theta_2 + \frac{m_1 l_1^2 + m_2 l_2^2}{4} + m_2 l_1^2)\ddot{\theta}_1 + \\
&\quad (I_2 + \frac{m_2 l_2^2}{4} + \frac{m_2 l_1 l_2}{2}\cos\theta_2)\ddot{\theta}_2 - \\
&\quad \frac{m_2 l_1 l_2}{2}\dot{\theta}_2^2 \sin\theta_2 - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2 \\
\tau_2 &= (I_2 + \frac{m_2 l_1 l_2}{2}\cos\theta_2 + \frac{m_2 l_2^2}{4})\ddot{\theta}_1 + \\
&\quad (I_2 + \frac{m_2 l_2^2}{4})\ddot{\theta}_2 + \frac{m_2 l_1 l_2}{2}\dot{\theta}_1^2 \sin\theta_2
\end{aligned}
$$

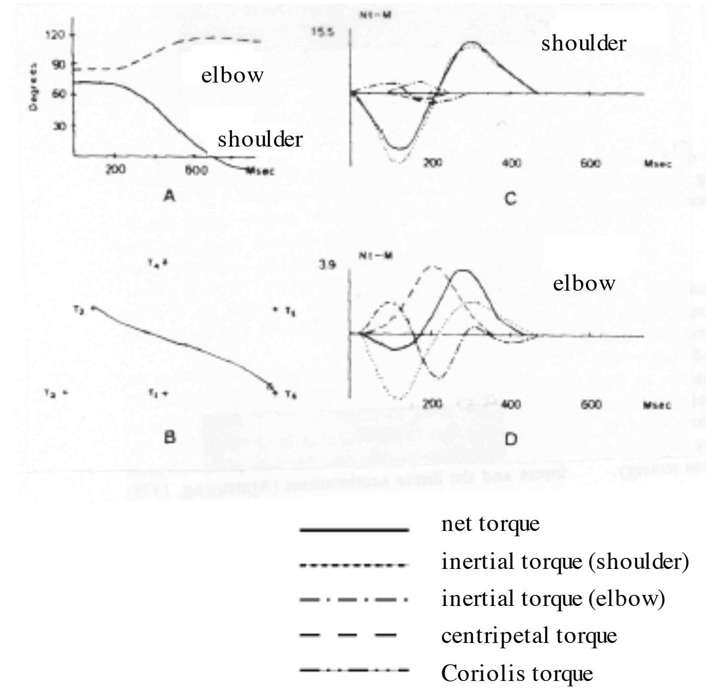


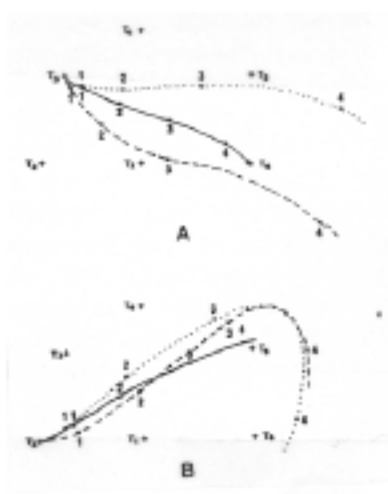
where τ_1, τ_2 are shoulder and elbow torques, m_1, m_2 segment masses, l_1, l_2 segment lengths, and I_1, I_2 moments of inertia.

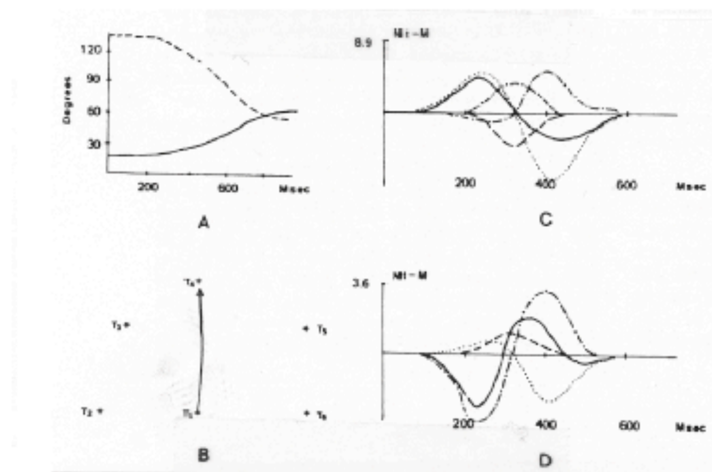
- **Inertial torques** : proportional to angular accelerations
  - Normal ($\tau_1 = \ldots + [] \times \ddot{\theta}_1 + \ldots$)
  - Interaction ($\tau_1 = \ldots + [] \times \ddot{\theta}_2 + \ldots$)
- **Velocity-dependent torques**
  - Coriolis ($\tau_1 = \ldots + [] \times \dot{\theta}_1\dot{\theta}_2 + \ldots$)
  - Centripetal ($\tau_1 = \ldots + [] \times \dot{\theta}_1^2 + \ldots$)
- **Gravity torques**

Mechanics (...)

Complex elbow torques: important contribution of inertial and centripetal torques due to shoulder displacements. Shoulder torques: close to torques during uniarticular movements.







Hollerbach & Flash (1982)

Muscle models

- 3 types of model (by complexity order)
  - Input/output model: black box that reproduces the behavior of a muscle in specific conditions. In general, linear transfer function that translates nervous signals into force.
  - Lumped model: combinaison of linear mechanic elements that reproduces the viscoelastic properties of muscles. Sometines nonlinear. Measurable parameters.
  - Cross-bridge model: description of molecular aspects of muscular contraction. Parameters not directly measurable.

- How to choose?
  - A more complex model requires a larger number of parameters.
  - What is expected influence of a complex model compared to a more simple one?

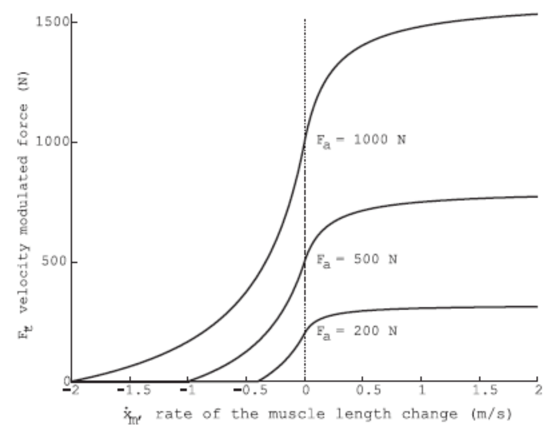
Lumped model

The muscle is made of 3 elements: (1) a contractile element (CE) which is a force generator; (2) a **serial** elastic element (SE) which represents the stiffness of tendon and cross-bridges acting in series with the force generator; (3) a **parallel** elastic element (PE) which represents the contribution of passive tissues.

Force/velocity relationship (Hill) $$V_{CE} = \frac{b(P_o - F)}{F + a}$$

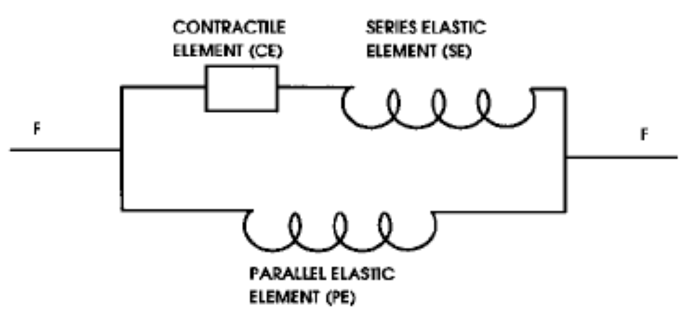
F Applied force

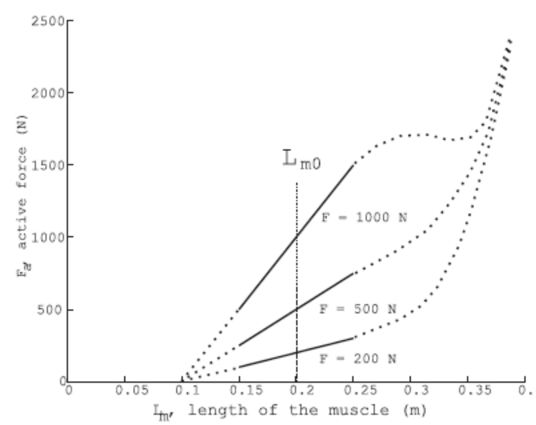
P_o Maximum isometric force



SE : force/length relationship (spring)

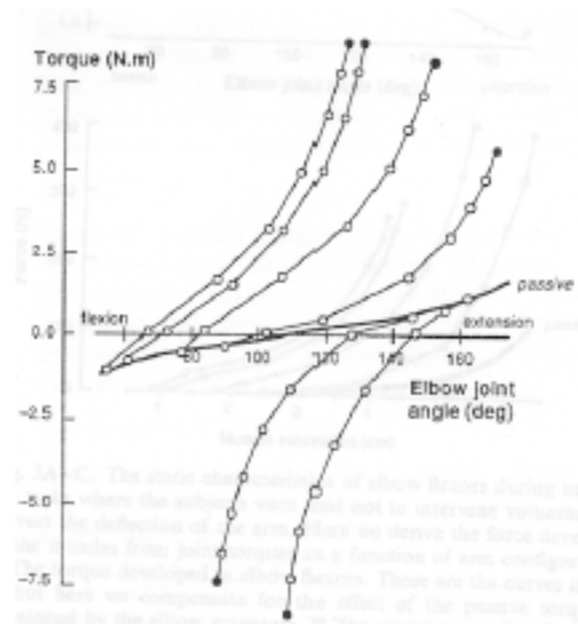
$$F_a = F + K(L_m - L_{m0})$$





Muscle + reflex

- Feldman’s experiment
  - Invariant characteristics. For supraspinal centers, the system muscle/reflex behaves as nonlinear spring with variable threshold length.



- 2 types of muscle
  - Variable threshold length

$$\phi = \Omega(\lambda, c_1, ..., c_n)$$

$$\phi = \Omega(\lambda - \beta)$$

$$\phi = k\left\{\exp\left[\alpha(\lambda - \beta)\right] - 1\right\}$$

λ muscle length

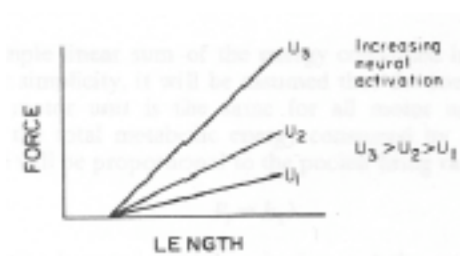
ϕ muscle force

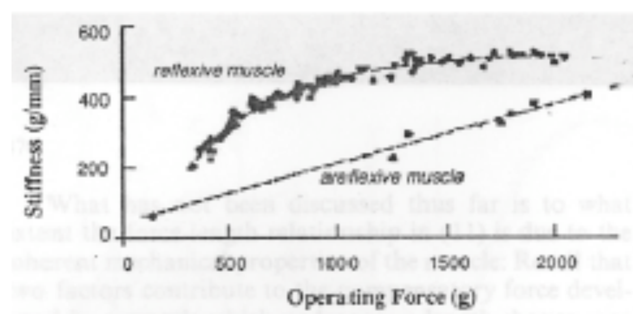
  - Variable stiffness

$$\phi = h(u)g(\lambda)$$

$$\frac{d\phi}{d\lambda} \propto \phi$$

u control





Stability

$$\tau = mc^2\frac{d^2\theta}{dt^2} + \nu\frac{d\theta}{dt} + mcg\cos\theta \qquad \lambda_{1,2} = \sqrt{b^2 + c^2 \pm 2bc\cos\theta}$$

$$\tau = -\mathbf{J}_M\phi = -\frac{d\lambda_1}{d\theta}\phi_1 - \frac{d\lambda_2}{d\theta}\phi_2$$

$$F(\theta,\omega) = \begin{cases} \dot\theta = \omega \\ \dot\omega = \frac{\tau}{mc^2} - \frac{\nu}{mc^2}\omega - \frac{g}{c}\cos\theta \end{cases}$$

$$\mathbf{K}_J = \frac{d\tau}{d\theta}$$

$$\mathbf{K}_J = -\left(\frac{d\mathbf{J}_M}{d\theta}\right)^T\phi - \mathbf{J}_M^T\frac{d\phi}{d\theta} = -\frac{d^2\lambda_1}{d\theta^2}\phi_1 - \left(\frac{d\lambda_1}{d\theta}\right)^2\frac{d\phi_1}{d\lambda_1} - \left(\frac{d\lambda_2}{d\theta}\right)^2\frac{d\phi_2}{d\lambda_1}$$

Shadmehr & Arbib (1992)

Equilibrium $(\dot\theta = \dot\omega = 0)$: $\theta_{eq} = \cos^{-1}(\tau/mgc) \qquad \omega_{eq} = 0$

Local linearization :

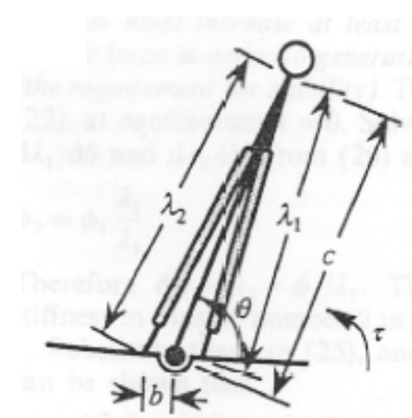
$$\frac{d}{dt}\begin{pmatrix}\theta \\ \omega\end{pmatrix} = F(\theta_{eq},\omega_{eq}) + \left.\frac{dF}{d(\theta,\omega)}\right|_{\theta=\theta_{eq},\omega=\omega_{eq}}\begin{pmatrix}\theta-\theta_{eq} \\ \omega\end{pmatrix} + ..$$

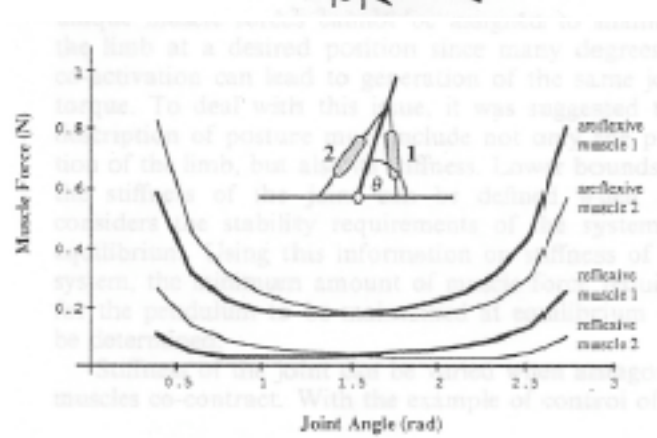
$$= \begin{pmatrix} 0 & 1 \\ \frac{d\tau}{d\theta}\frac{1}{mc^2} + \sin\theta_{eq} & -\frac{\nu}{mc^2}\end{pmatrix}\begin{pmatrix}\theta-\theta_{eq} \\ \omega\end{pmatrix} + \ldots$$

$$e_{1,2} = \frac{1}{2}\left\{-\frac{\nu}{mc^2} \pm \left[\frac{\nu^2}{m^2c^4} + 4\left(\frac{K_J}{mc^2} + \frac{g}{c}\sin\theta_{eq}\right)\right]^{1/2}\right\}$$

Stability condition : $K_J < -mcg\sin\theta_{eq}$

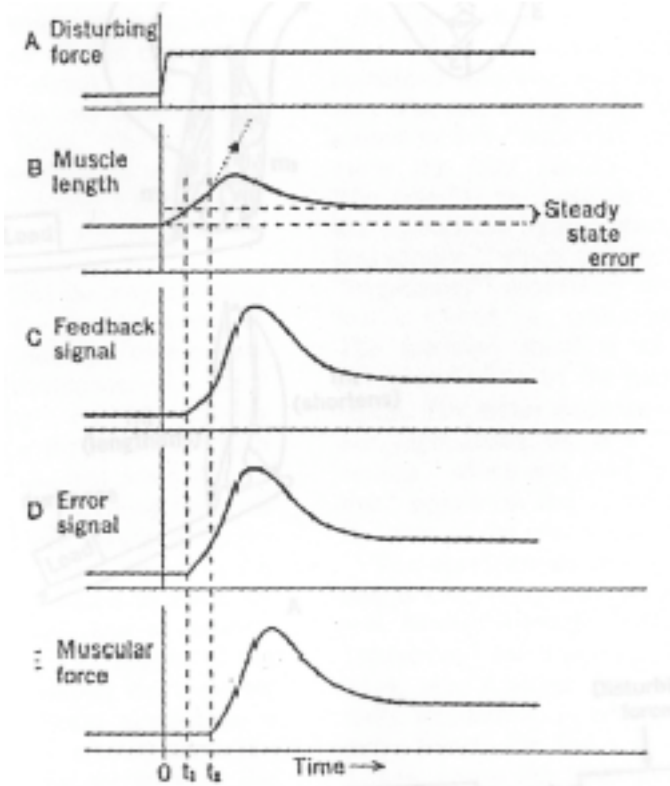
$$K_J = \frac{b^2c^2\sin^2\theta}{\lambda_1^3}\left(\phi_1 - \lambda_1\frac{d\phi_1}{d\lambda_1}\right) \qquad \frac{d\phi_1}{d\lambda_1} > \frac{\phi_1}{\lambda_1}$$

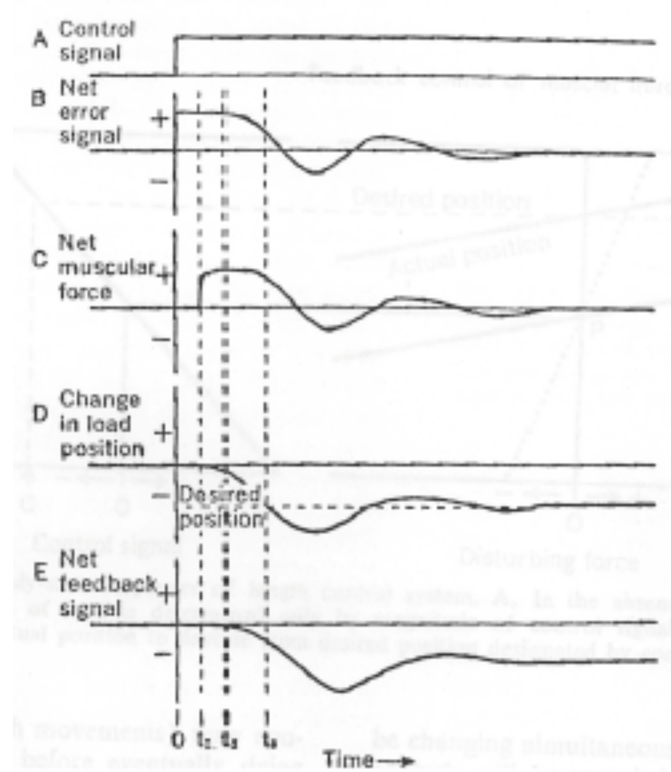


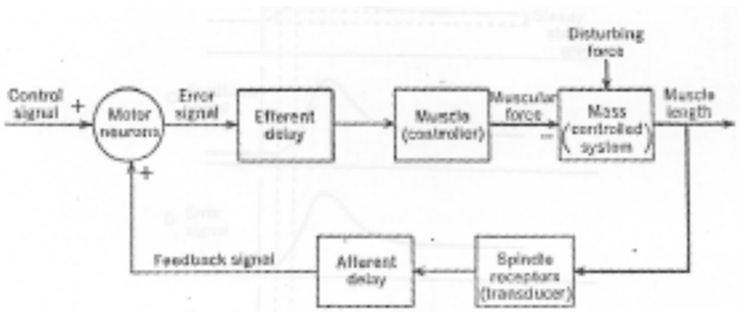


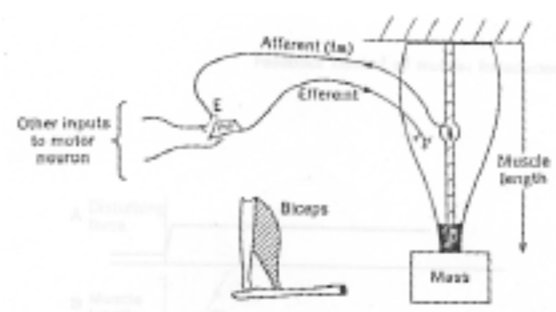
Stiffness should increase at least linearly with force.

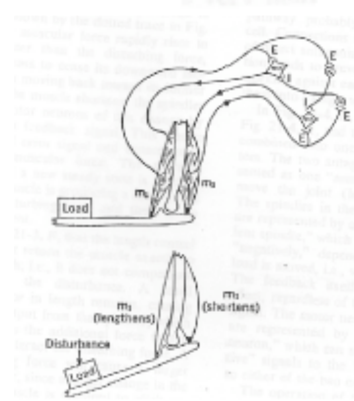
Functional role



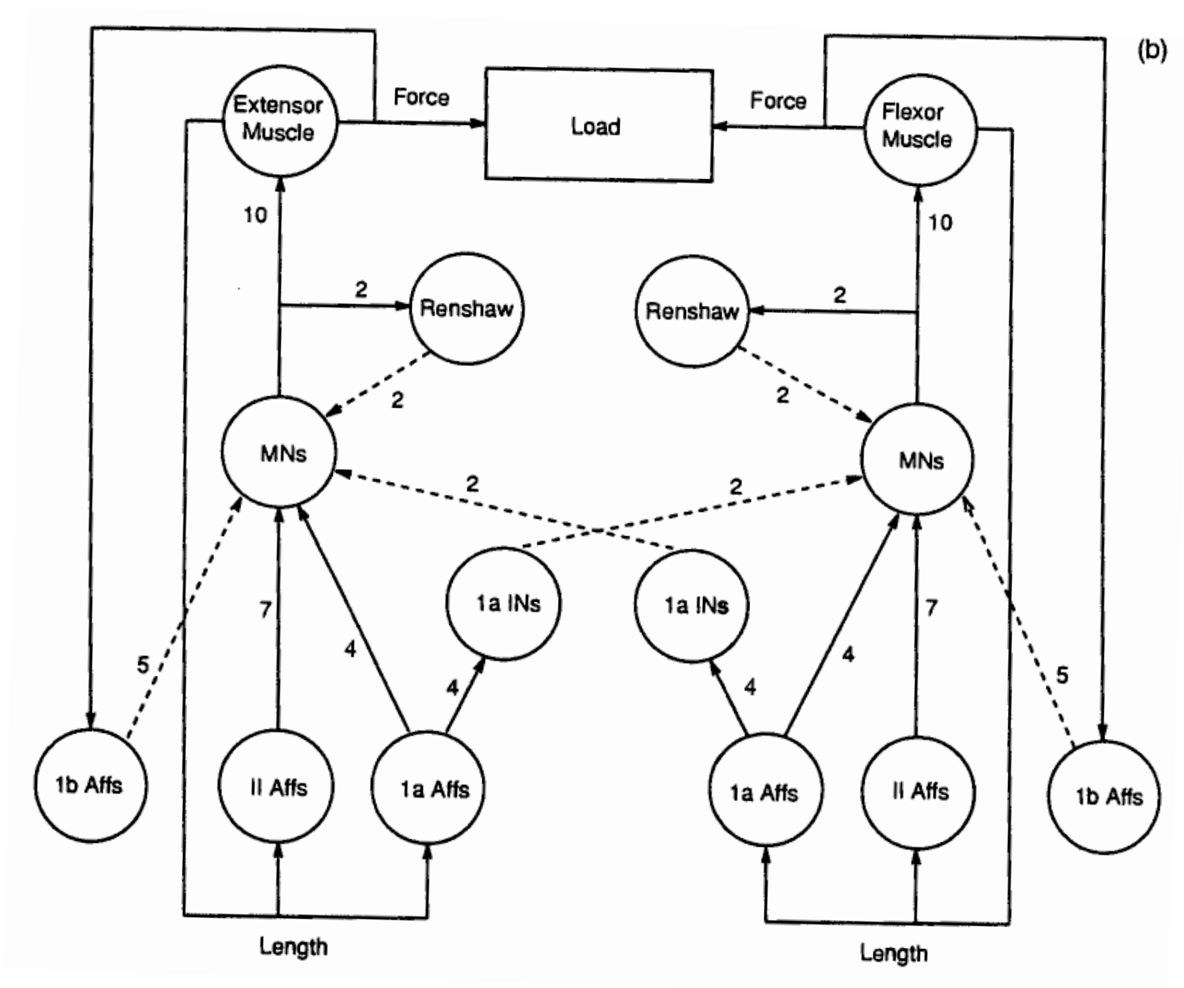


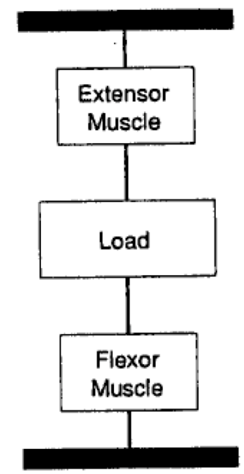






Model of spinal circuits

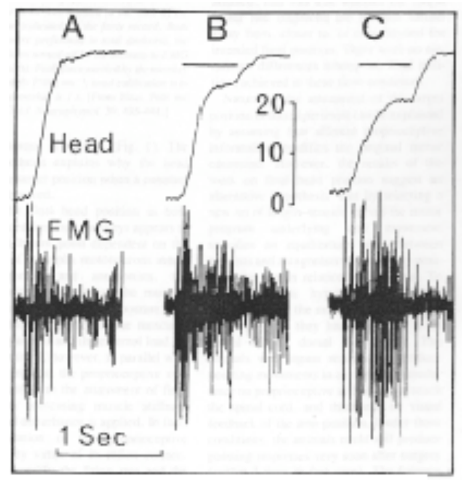


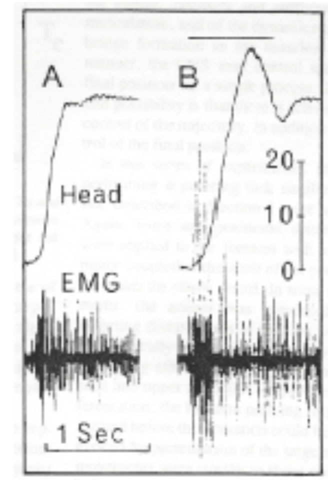


Graham & Redman (1993)

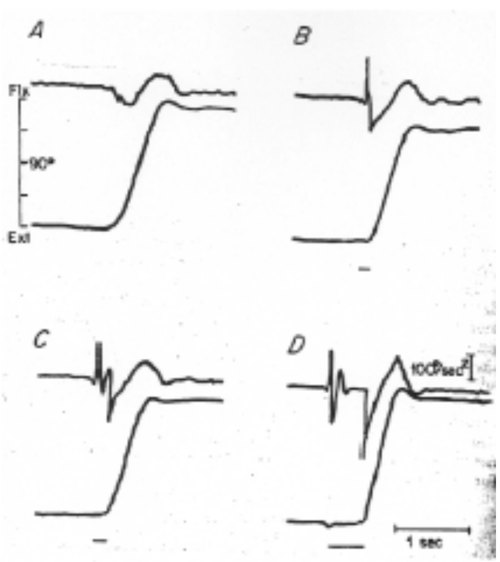
Equilibrium point theory

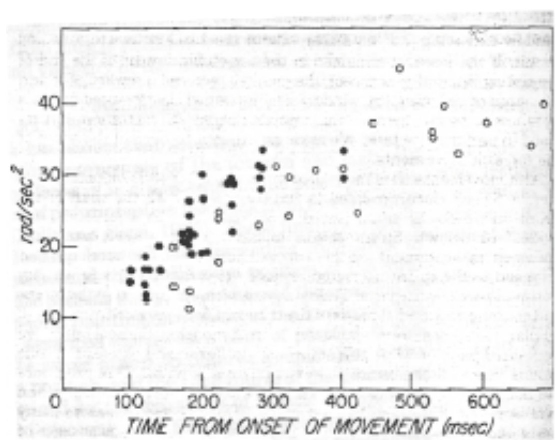
Experimentally. Hypothesis of final position control: the nervous system controls a movement by specifying the final equilibrium position of the limb. The characteristics of the actual trajectory reflect inertial and viscoelastic properties of the limb and neuromuscular system.



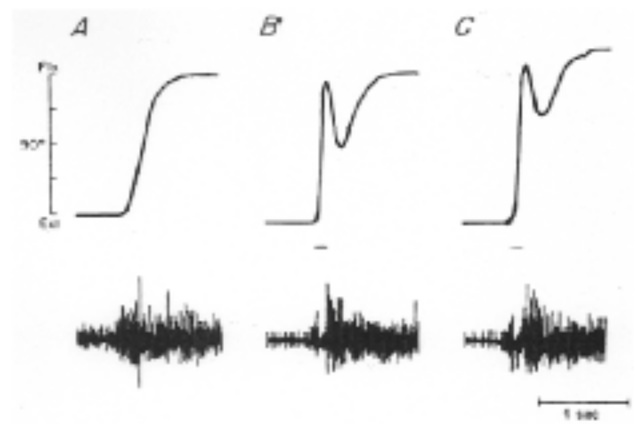


Bizzi et al. (1976)



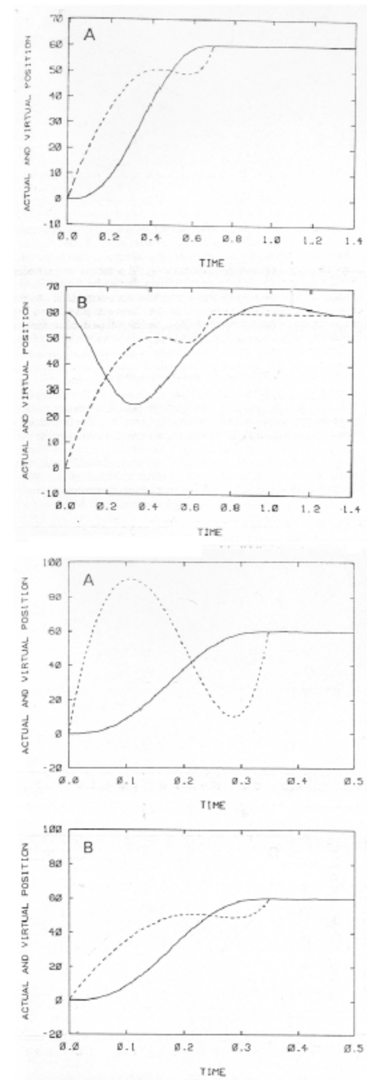


Bizzi et al. (1984)



Equilibrium trajectory

Study of single-joint movements (forearm). At a given time, the muscular activation represent an equilibrium position of the segment. The variation in muscular activations describes an **equilibrium trajectory** (or virtual trajectory). If the segment moves, the virtual position defined by the muscular activations can be different from the real position. The virtual position is the position toward which the current muscular activations displace the segment.



Hogan (1984)

$$I\ddot{\theta} = T(\theta, \dot{\theta}, \{a\})$$

$$\text{Equilibre} : T(\theta, \{a\}) = 0$$

$$\theta_0 = \theta_0(\{a\})$$

$$T(\theta, \dot{\theta}, \{a\}) = T(\{a\}) - K\theta - B\dot{\theta}$$

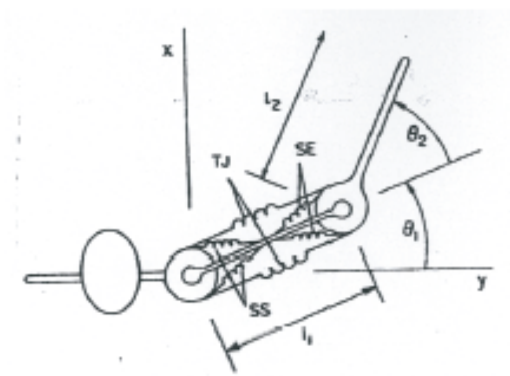
$$T(\{a\}) = K\theta$$

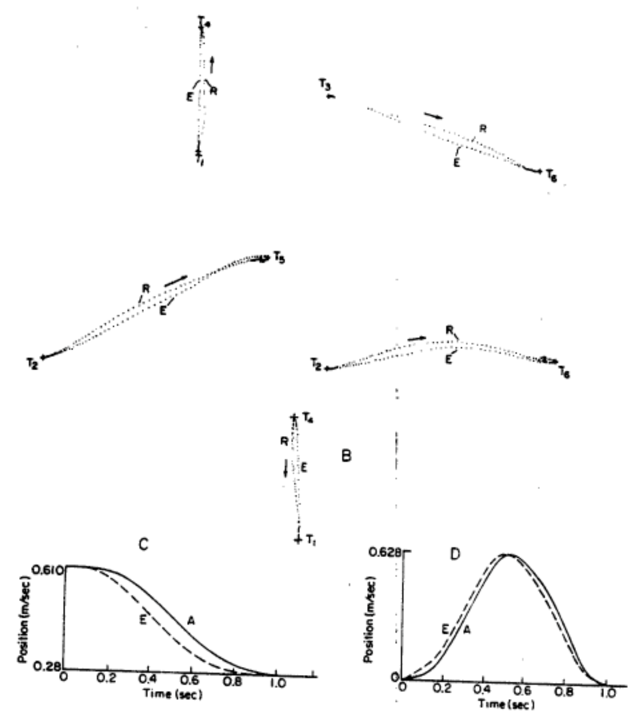
$$\theta_0(\{a\}) = T(\{a\})/K$$

$$I\ddot{\theta} + B\dot{\theta} + K\vartheta = K\theta_0(\{a\})$$

Equilibrium trajectory (...)

Calculus of equilibrium trajectories from actual trajectories for a multi-articular system.





$$\mathbf{n} = \mathbf{I}(\theta)\ddot{\theta} + \mathbf{C}(\dot{\theta}, \theta)\dot{\theta}$$

$$\mathbf{n}(t) = \mathbf{R}(t)\,[\phi(t) - \theta(t)] - \mathbf{B}(t)\dot{\theta},$$

$$\mathbf{R} = \begin{pmatrix} R11 & R12 \\ R21 & R22 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} B11 & B12 \\ B21 & B22 \end{pmatrix}$$

$\mathbf{n}$ torques

$\mathbf{I}$ inertia matrix

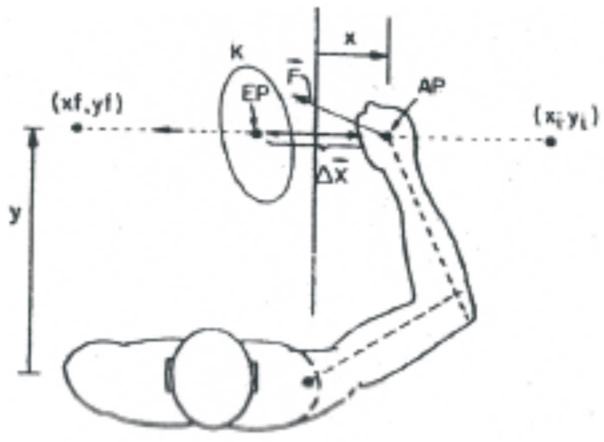
$\mathbf{C}$ matrix of velocity-dependent forces

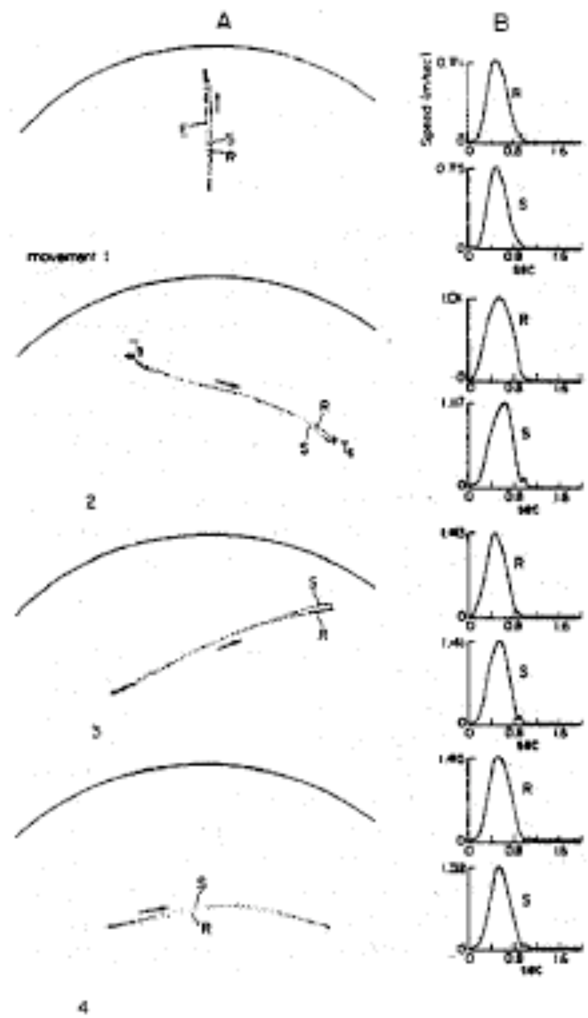
ϕ vector of equilibrium angles

Flash (1987)

Equilibrium trajectory (...)

Calculus of actual trajectories for a given equilibrium trajectory.



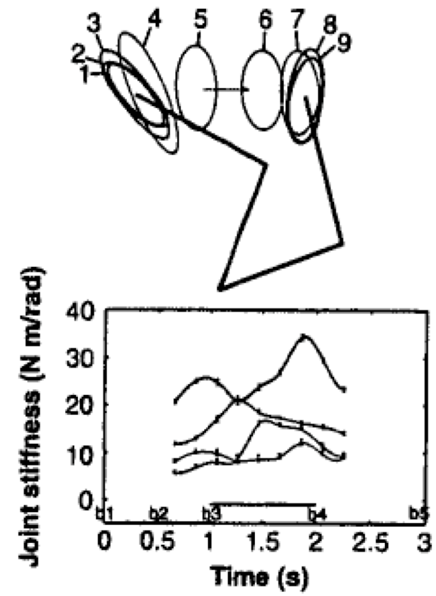


!! Fast movements require a larger stiffness and viscosity. For 0.5-0.8 s movements, calculated trajectories are close to real trajectories. Below 0.5 s, differences are observed. The scaling strategy is not uniform. Some movements require a change in the shape and orientation of stiffness and viscosity ellipses.

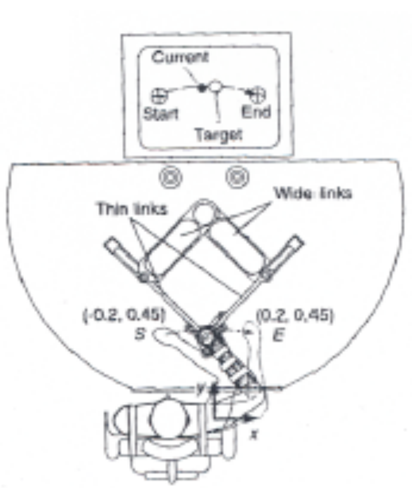
Flash (1987)

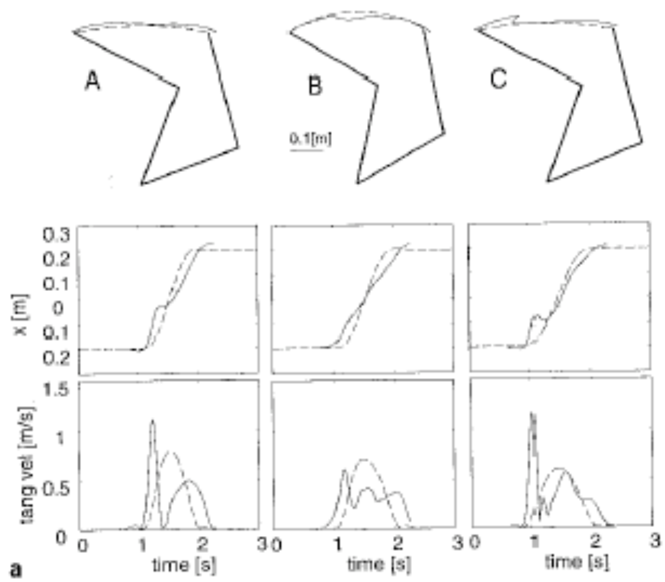
Difficulty: Stiffness

Hypotheses for the equilibrium point theory: (1) Elastic properties of the neuromuscular system are exploited for motor control; (2) The nervous system uses a virtual trajectory as a descending command; (3) The virtual trajectory is easy to construct; it is unnecessary to solve the inverse dynamics problem for the controlled object.



$$\mathbf{q}_{eq} = \mathbf{R}^{-1}(\tau_{in} + \mathbf{D}\dot{\mathbf{q}}) + \mathbf{q}$$
$$\tau_{in} = \mathbf{R}(\mathbf{q}_{eq} - \mathbf{q}) - \mathbf{D}\dot{\mathbf{q}}$$





Gomi & Kawato (1996)

Dynamic stiffness is not large enough to obtain a virtual trajectory close to the actual trajectory.

Difficulty: Solution

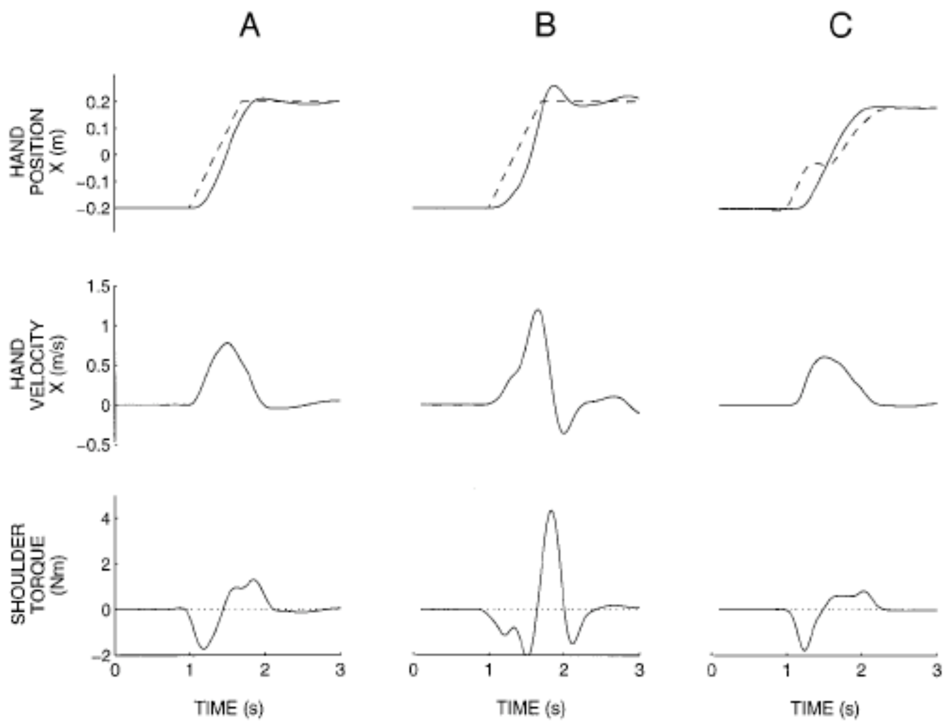
Nonlinear muscle model

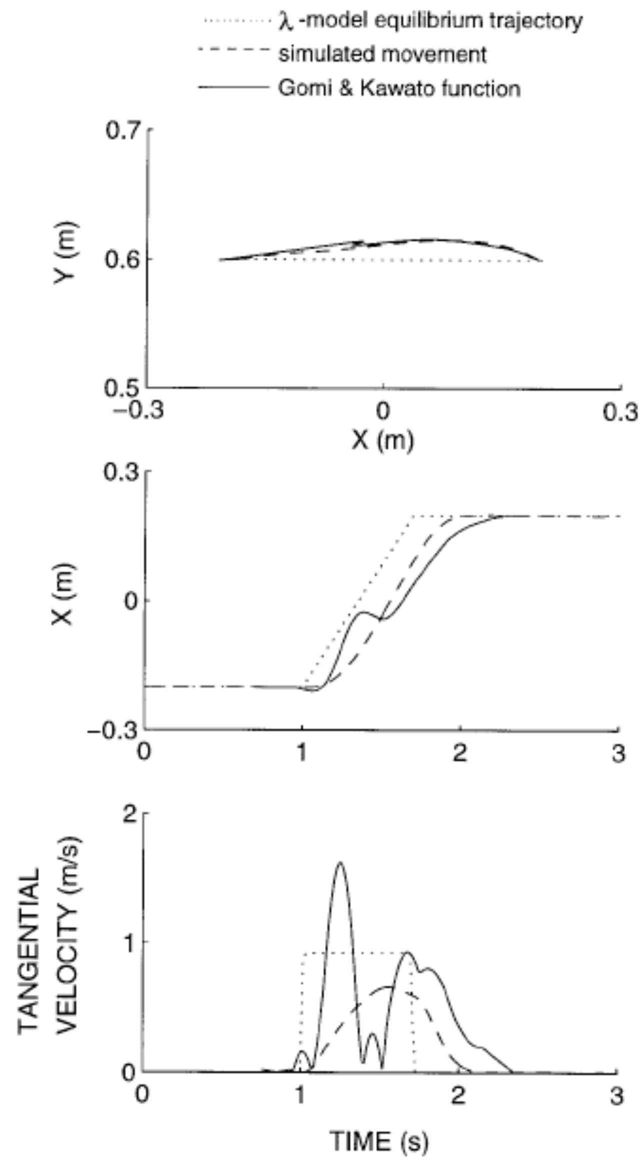
$$A = [l - \lambda + \mu \dot{l}]^{+}$$

$$\tilde{M} = \rho\,[\exp(cA) - 1]$$

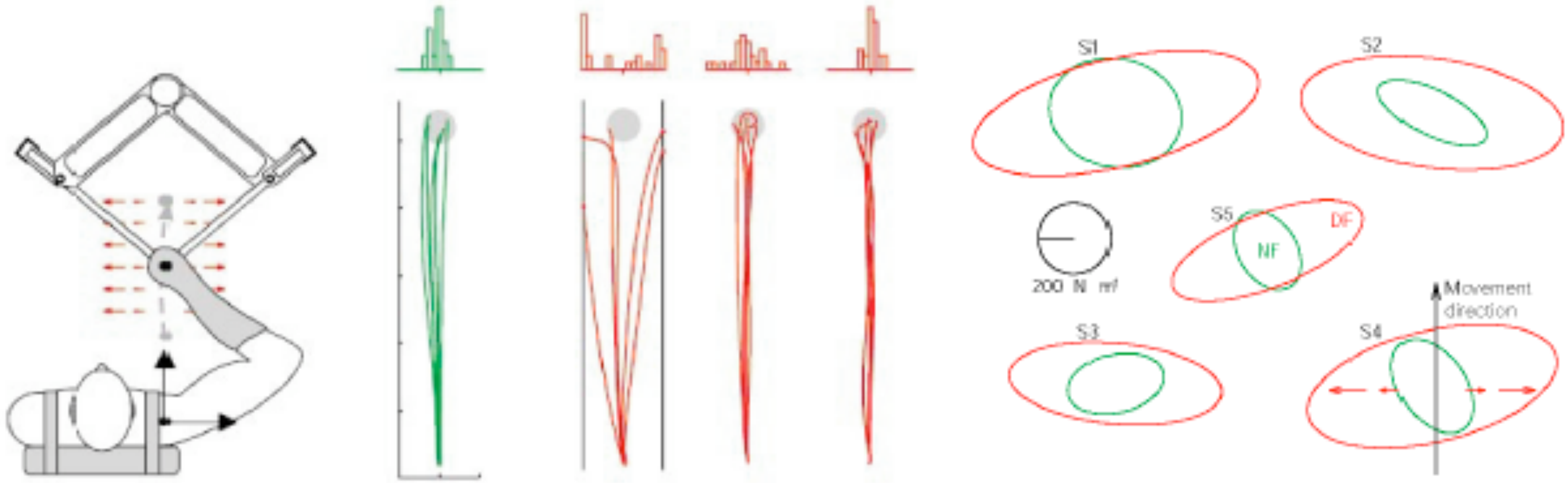
$$\tau^2 \ddot{M} + 2\tau \dot{M} + M = \tilde{M}$$

Gribble et al. (1998)

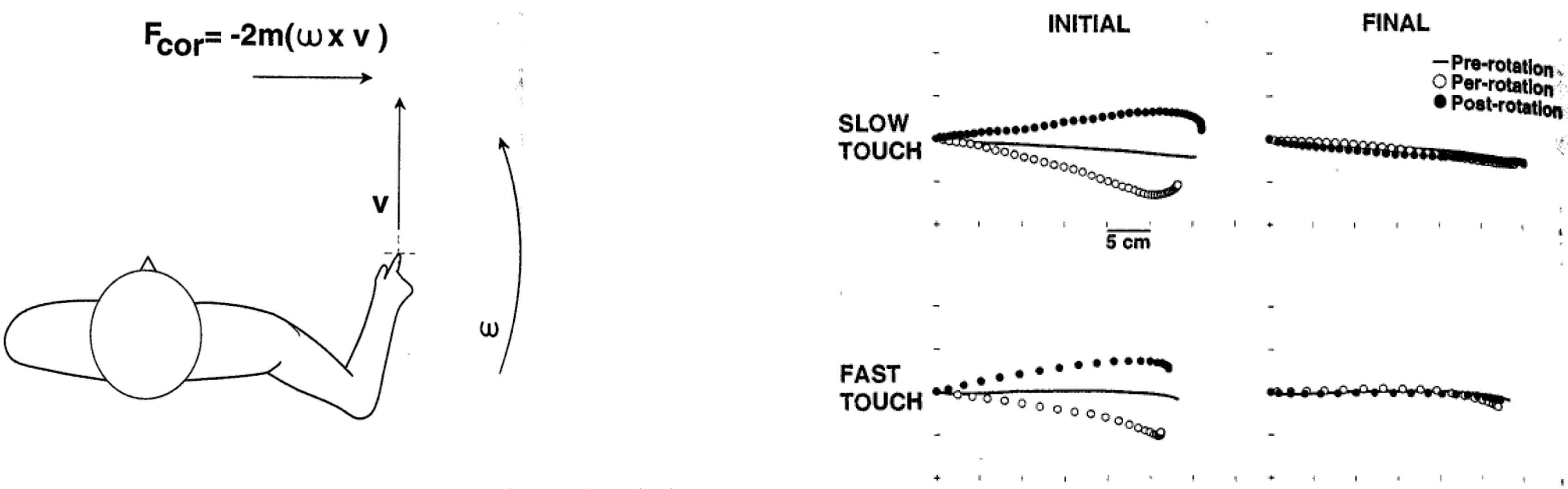




More difficult



Burdet et al. (2001)



Lackner & DiZio (1994)

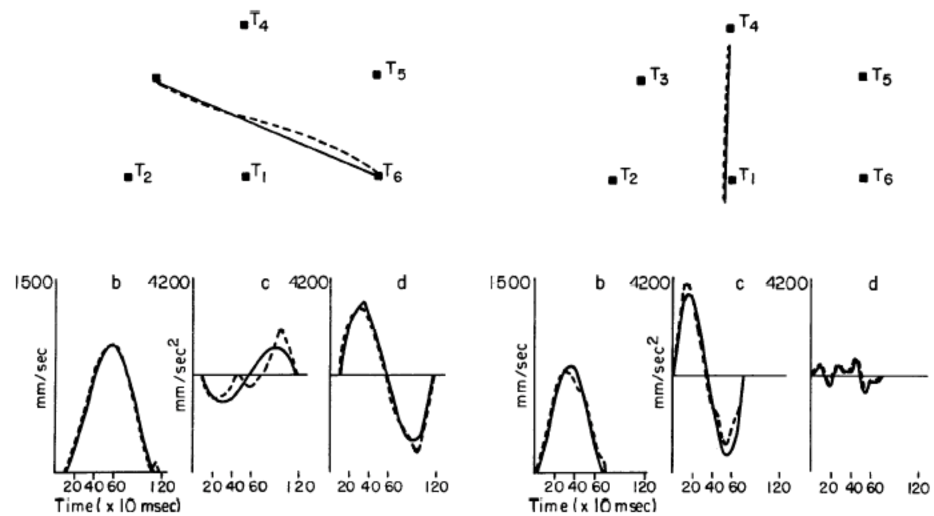
Optimal control: Kinematics

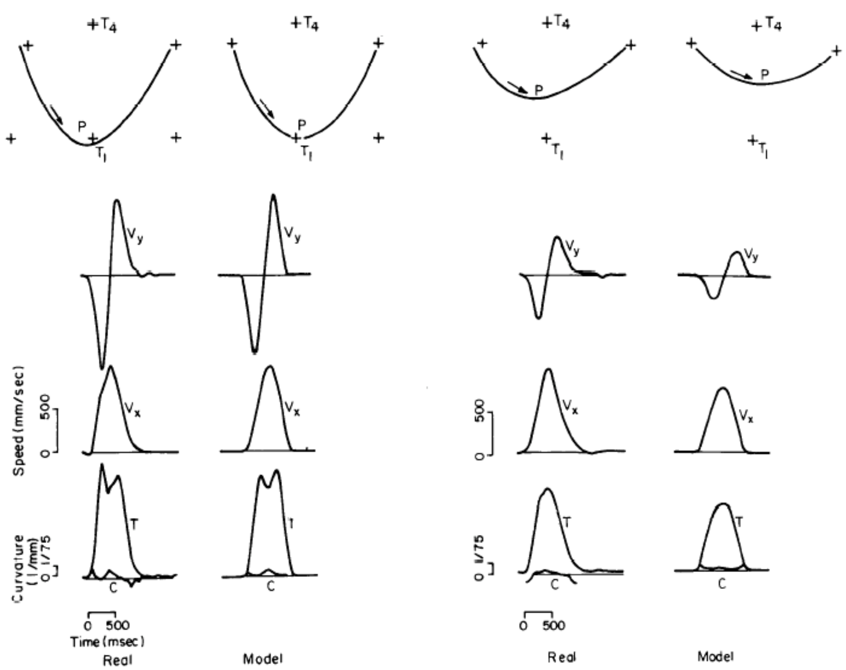
Minimum-jerk

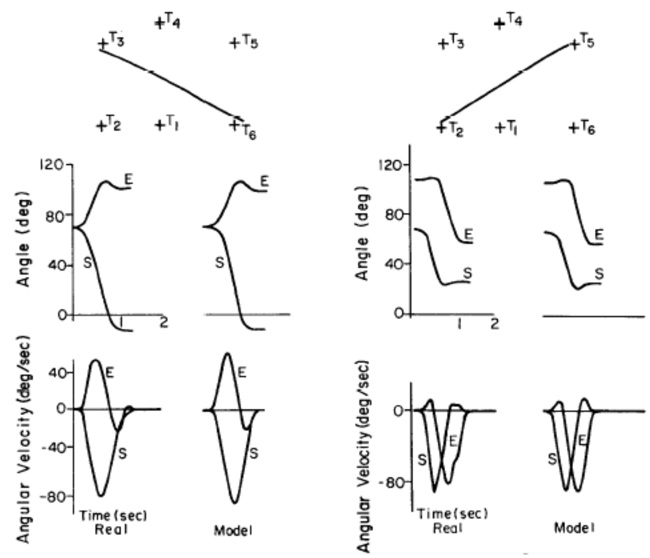
$$C = \frac{1}{2}\int_0^{t_f}\left(\left(\frac{d^3x}{dt^3}\right)^2 + \left(\frac{d^3y}{dt^3}\right)^2\right)dt$$

$$x(t) = x_0 + (x_0 - x_f)(15\tau^4 - 6\tau^5 - 10\tau^3)$$
$$y(t) = y_0 + (y_0 - y_f)(15\tau^4 - 6\tau^5 - 10\tau^3)$$

Flash & Hogan (1985)



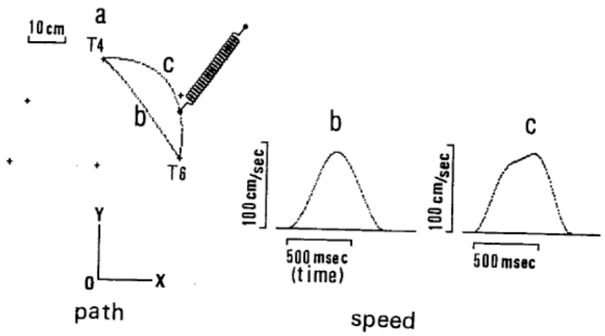




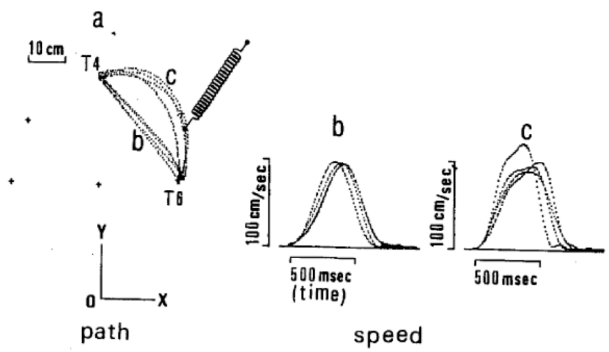
Optimal control: Dynamics

Minimum-torque change

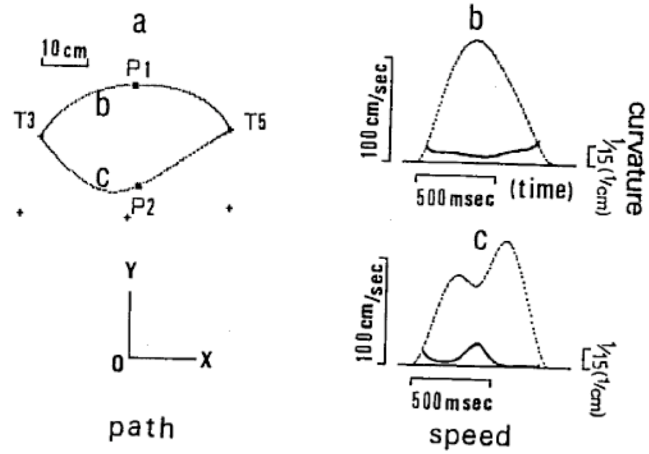
$$C = \int_{t_0}^{t_f} \sum_i \left(\frac{d\tau_i}{dt} \right)^2 \, dt$$



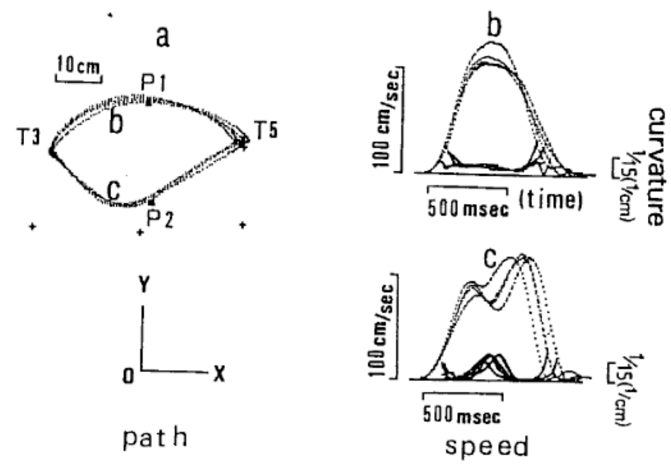
A



B



A



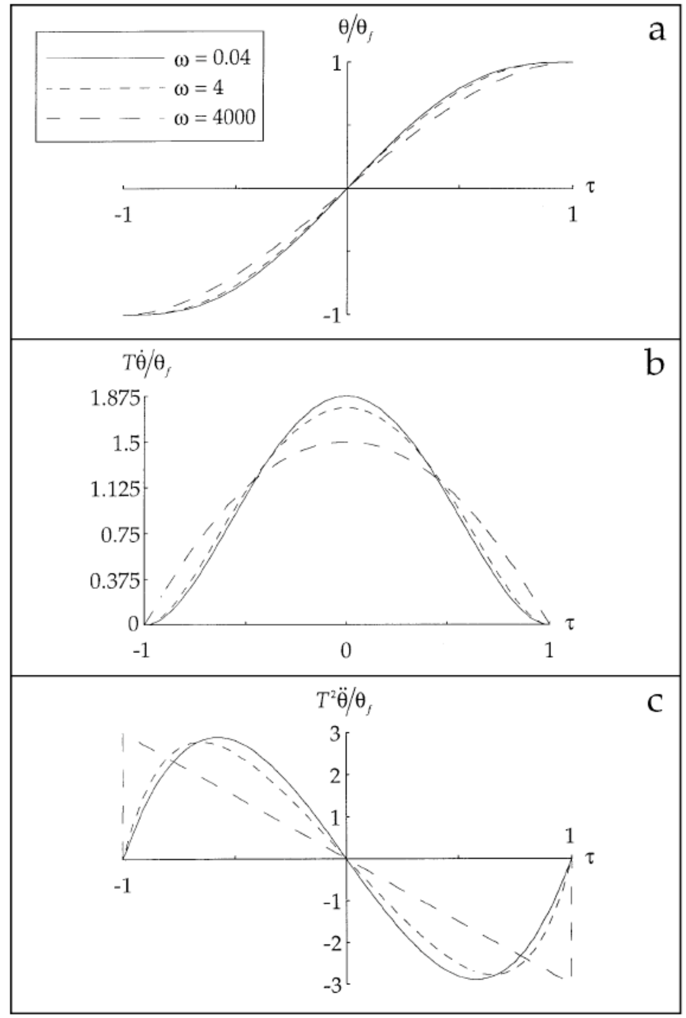
B

Uno et al. (1989)

Minimum-torque change

Analytic study in the case of uniarticular movements: (1) trajectories have a unique peak velocity at half movement time (MT); (2) the ratio R of peak velocity to mean velocity is within [1.5;1.875].

Incompatible with data on forearm flexion movements. Slow movements have their peak velocity before MT/2, and fast movements after MT/2 (e.g. (0.58±0.03)MT for very fast movements). Mesured R is 1.77-1.89 for slow movements, and 2.01-2.09 for fast movements.



Engelbrecht & Fernandez (1997)

$$\tau = I\ddot{\theta} + B\dot{\theta}$$

$$C = \int_{t_0}^{t_f} \dot{\tau}^2 \, dt$$

$$\frac{d^3}{dt^3}\frac{\partial \dot{\tau}^2}{\partial \dddot{\theta}} - \frac{d^2}{dt^2}\frac{\partial \dot{\tau}^2}{\partial \ddot{\theta}}$$

$$\beta = B/I$$

$$\omega = \beta(t_f - t_0)$$

$$\theta^{(6)}(t) - \beta\theta^{(4)}(t) = 0$$

$$\theta(t) = u_1 \sinh(\beta t) + u_2 \cosh(\beta t) + u_3 t^3 + u_4 t^4 + u_5 t^5 + u_6$$

Other cost functions

- Energy, effort, force, force change, duration, ...

- No function appears to be really superior (e.g. by making better predictions).

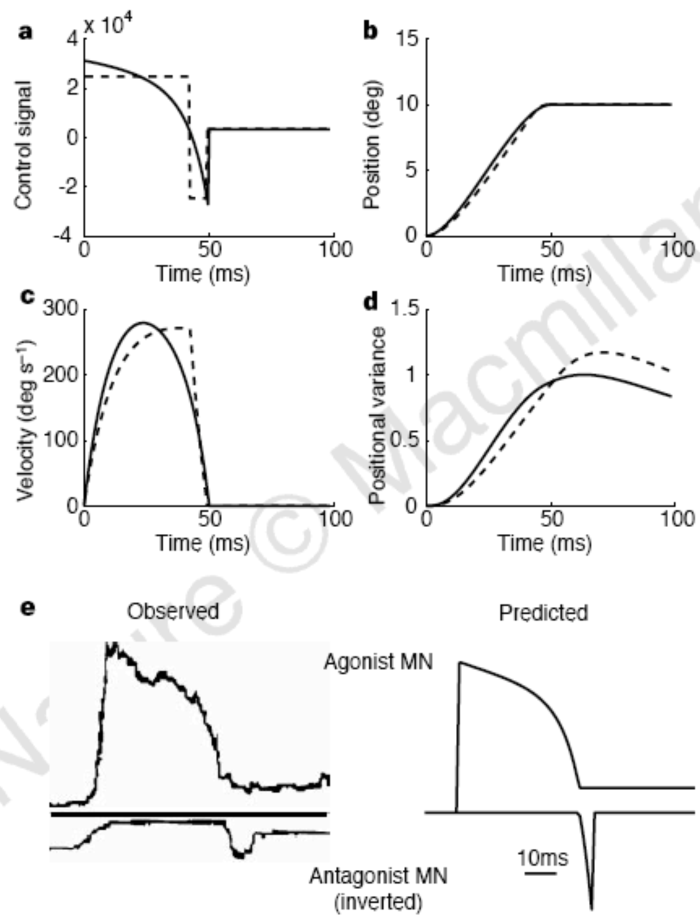
- Arbitrary nature of cost functions.

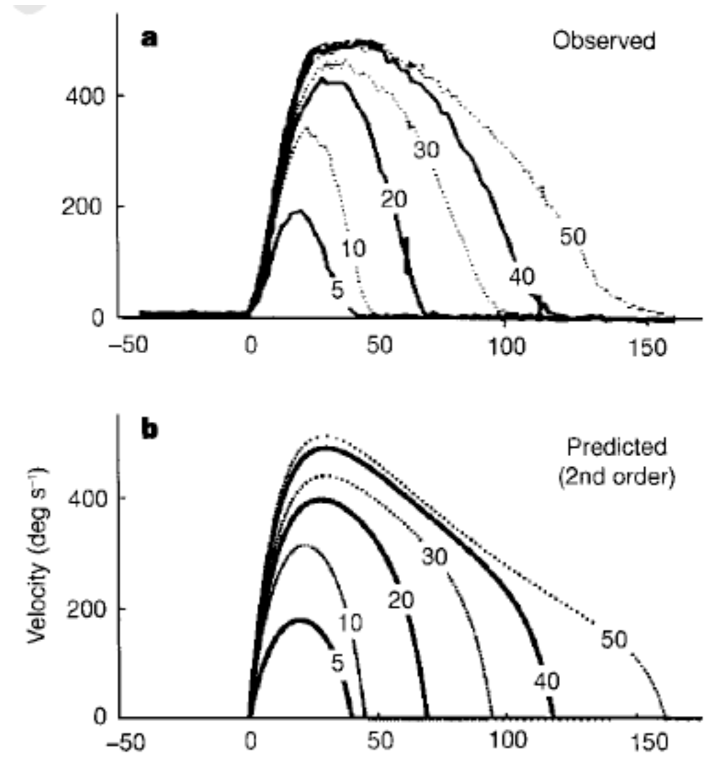
- No underlying principles.

- How can the nervous system measure of a cost?

Minimum variance

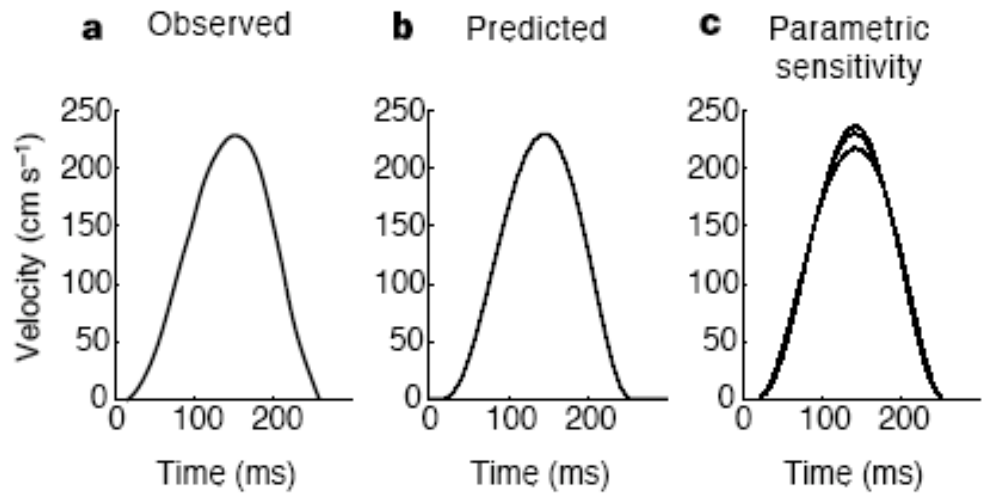
Minimize the terminal variance in the presence of noise. SDN (signal-dependent noise): the variance of noise increases with the size of the command. In fact : **minimum variance = smallest command**



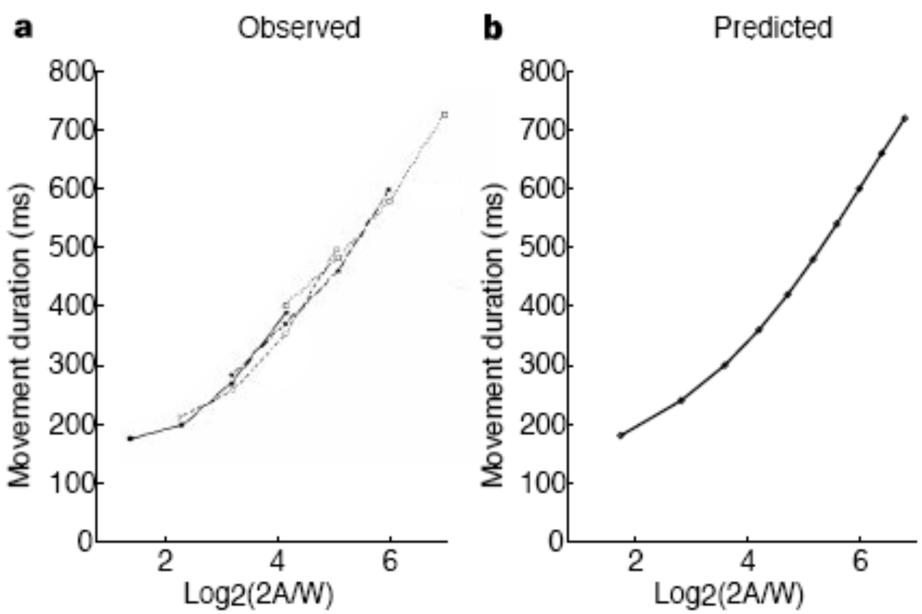


Harris & Wolpert (1998)

Minimum variance (...)



Fitts' law



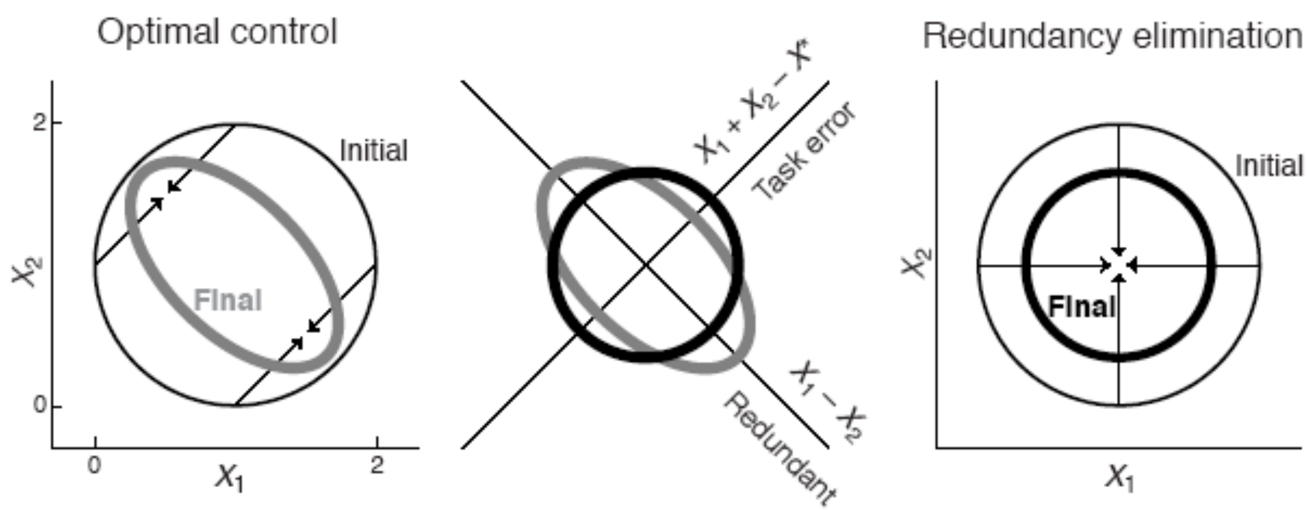
!! Open loop model

Harris & Wolpert (1998)

Stochastic optimal feedback control

• Paradox: ability to reach a goal in a fiable and repetive way vs. variability of each trial.

• « Uncontrolled manifold »: fluctuations on individual dof are larger than on the parameters to be controlled (i.e. specified by the task). Variability is constrained to a redundant subspace rather than being suppressed in a nonspecific manner.

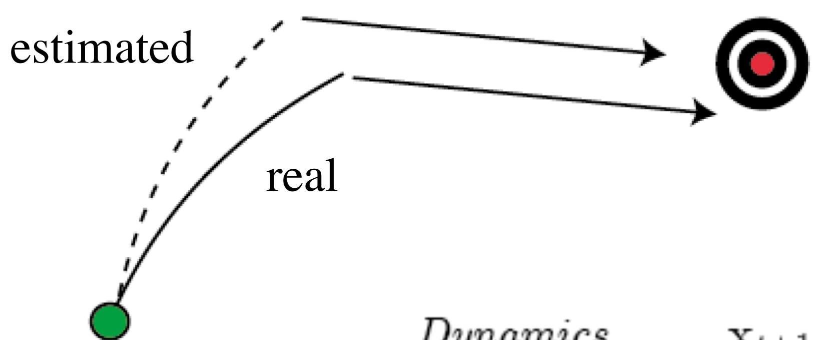


Todorov & Jordan (2002)

SOFC (...)

- Following a trajectory vs. reaching a goal.
- Planification/execution vs. online control.
- Control

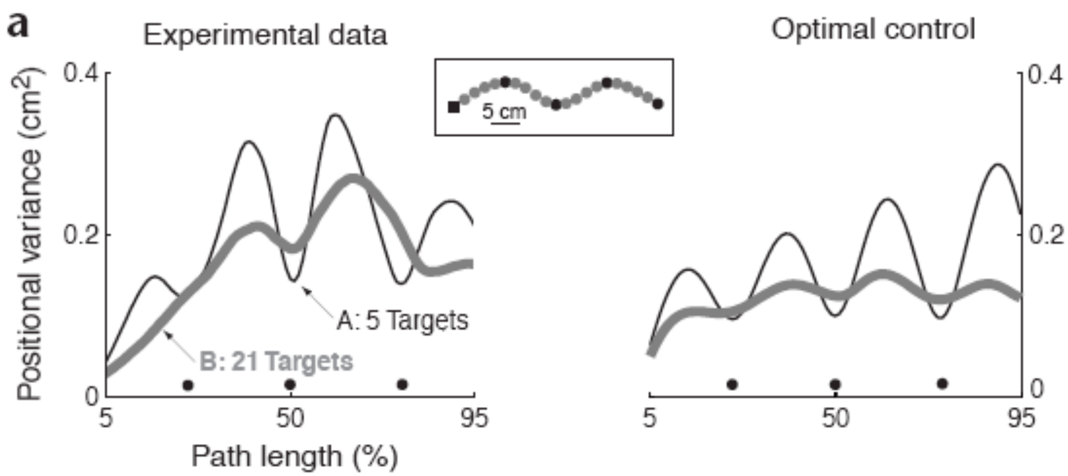
  optimal: minimum error and effort
  feedback: optimal reprogramming at each time
  stochastic: taking the statistics of noise into account

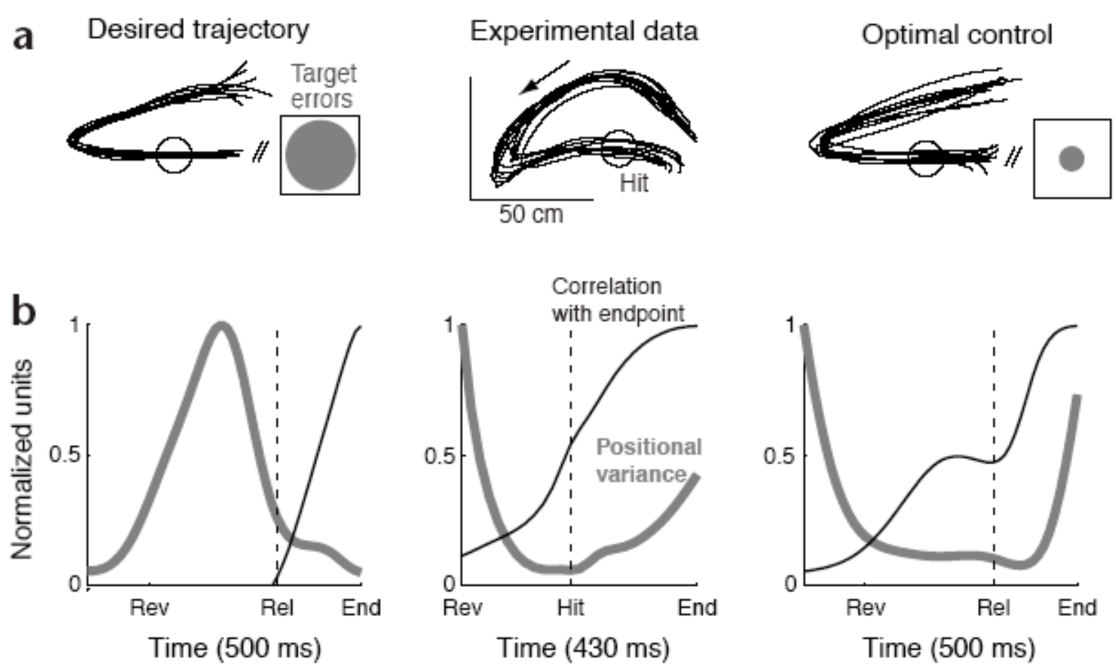


Todorov & Jordan (2002)

Dynamics	$\mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t + \xi_t + \sum_{i=1}^{c} \varepsilon_t^i C_i \mathbf{u}_t$
Feedback	$\mathbf{y}_t = H\mathbf{x}_t + \omega_t + \sum_{i=1}^{d} \epsilon_t^i D_i \mathbf{x}_t$
Cost per step	$\mathbf{x}_t^T Q_t \mathbf{x}_t + \mathbf{u}_t^T R \mathbf{u}_t$

SOFC (...)





Todorov & Jordan (2002)

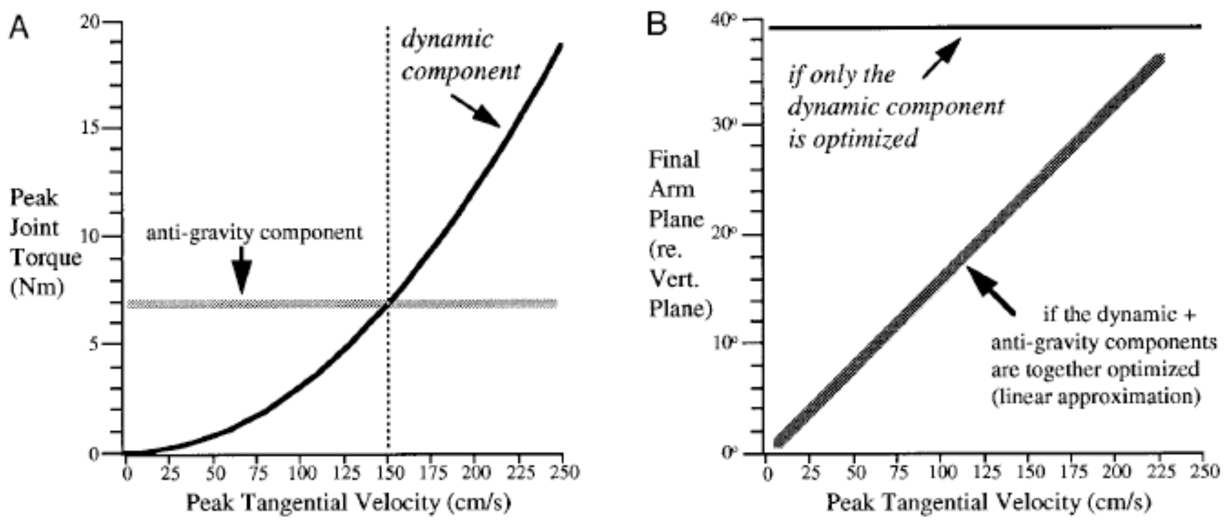
Limitations

• Motor noise: emergence of Fitts' law, but incompatible with the relationship between cocontraction and precision.

• Stochastic control.

• Cost function error/effort.

• Simultaneous control of posture and movement.



Nishikawa et al. (1999)