

## Time optimality in the control of human movements

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**Abstract.** In a simulation study the control of maximally fast goal directed movements has been analyzed. For a simple linear model it is shown that the presence of a third input block reduces the movement duration. The time optimal size of the third block depends on the ratio of a neuromuscular time constant (first-order lag) and movement time. As a second step a non-linear muscle mode was simulated. By an optimization of input parameters it was found that the time optimal input, as expected, switches between maximal agonist and maximal antagonist activation. As for the linear model, a third phase was required for an optimal movement. It was found that the third phase serves to compensate the slowly decaying antagonist force. Also an input similar to experimentally found activation patterns was simulated. This input contains a silent period between the first two bursts and the second and the third burst have submaximal amplitudes. This input led to a near time optimal movement with a duration 9% larger than the minimal duration but with largely reduced muscle forces. This suggests that a criterion is minimized which also takes into account the effort spent. Including gravity in the model indicates optimality of a silent period between the third phase and a final agonist activity to resist gravity. When assuming different dynamics for agonist and antagonist, the optimal switch times for agonist and antagonist no longer coincide, also after the three block pattern some extra activity is required to obtain a cancellation of the slowly decaying force in agonist and antagonist.

### Introduction

In the literature various attempts have been made to describe the control of human movements as the result of an optimization process; it is then assumed that muscular inputs are chosen such that a maximal or satisfactory performance is obtained at a minimal or acceptable cost. Performance measures are for instance impulse in jumping or mechanical work in cycling.

Costs mostly relate to muscle fatigue. Finding performance and/or cost criteria which adequately describe motor behavior for certain tasks can be of value to:

a) Support a description of learning in terms of optimization;

b) given a certain task, enable computation of an optimal control and trajectory which can then be regarded as a norm for the behavior of healthy trained subjects;

c) enable estimation of inputs and muscle forces given a certain movement trajectory (solve the inverse dynamic problem).

The current contribution concerns maximally fast goal directed movements. This implies time optimality or a minimization of the movement duration.

In a previous contribution (Happee and Daanen 1991a, b) goal directed arm movements were studied. The arm was held in the sagittal plane, the hand was moved forward or backward, the forearm stayed roughly horizontal and the upper arm ranged between about 20 deg anteflexion and 15 deg retroflexion (see Ruitenbeek and Jansen 1984 for a description of the setup). In these movements, the anteflexors and retroflexors of the shoulder play a major role in the acceleration and deceleration of the limb. In the EMG of maximally fast movements, triphasic activity was found comparable to activity reported by others (Wadman et al. 1979; Gielen et al. 1984, 1985; Marsden et al. 1983; Flament et al. 1984). The agonist activity in the first phase causes acceleration of the limb. The antagonist activity in the second phase serves to decelerate the limb. The agonist activity in the third phase is shown to be relevant in fast movements (Hannaford and Stark 1985, 1987; Wierzbicka et al. 1986). However, why the third phase is relevant is not clear; Hannaford and Stark (1987) state that it serves to "clamp" the movement but do not clarify what is meant by this.

According to the Pontryagin maximum principle (Mackay and Strauss 1982), generally for dynamic systems with bounded inputs, the time optimal control is a bang-bang control; inputs switch a number of times

between minimal and maximal values. This suggests switching between maximal agonist and maximal antagonist inputs. However, as agonist and antagonist inputs can be controlled separately, maximality only implies switching between zero and maximal input. Maximality does not imply that switching of agonists and antagonists should occur simultaneously.

The number of input switches of course determines the number of input blocks. From the linear control theory it is known that to transfer a system of order  $N$  to any desired state, generally it requires at most  $N-1$  switches between minimal and maximal value per input. These  $N-1$  switches do not include the switching from and to zero at the beginning and ending of the input sequence. When assuming maximality, the number of free parameters in the input sequence equals the sum of the number of switches over all inputs plus one for the duration of the sequence. To obtain reachability of the  $N$  dimensions of the state space, the number of parameters in the input sequence must be at least  $N$ . So, the sum of switches in all inputs must be at least  $N-1$ .

When we now assume a time optimal control of three maximal blocks we have 2 switches for the agonist and 2 for the antagonist. These values as expected lie below the maximum of 5 switches for a sixth-order model as used by Hannaford and Stark (1987). However, the 5 free parameters in the input sequence are not sufficient to control the 6 dimensions of the state space. This counts even more for the eighth-order model described by Winters and Stark (1985). This suggests that the time optimal control may even consist of more than three phases.

As shown in Fig. 1 the EMG patterns in our experiments differed from what can be expected (but is not proven) to be time optimal; significant differences in timing were found between agonist and antagonist EMG and submaximal amplitudes were found in the second and third phase. Also, in the EMG, a pre-movement antagonist activity was found and after the movement, agonist activity was found. This activity can be assumed to serve to resist gravity in a stationary position. Between the end of the pre-movement antagonist activity and the start of the first agonist burst a silent period of 31–93 ms was found. A silent period of 15–75 ms was found between the first agonist burst and the antagonist burst. Mostly insignificant silent periods were found between the second and the third burst. The

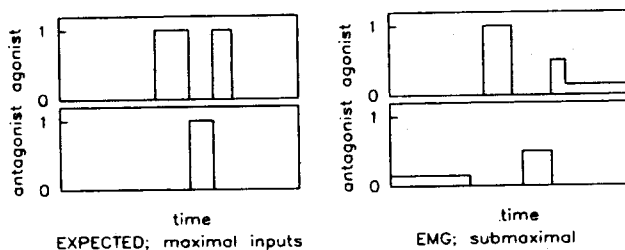


Fig. 1. Expected time optimal control with maximal amplitudes and synchronous switching of agonist and antagonist (*left*) and control matching EMG signals found (*right*)

silent periods found have considerable durations given the duration of the first agonist burst which ranged from 59–171 ms and the duration of the antagonist burst which ranged from 68–183 ms. Instead of a silent period, data of Gielen et al. (1985) show an overlap of the first agonist burst and the antagonist burst. This overlap may partly be caused by the performed ensemble averaging. Another explanation for the overlap reported is that the authors investigated movements with durations ranging from 100 to 250 ms, where in our experiments subjects could not obtain movement durations below 200 ms. Marsden et al. (1983) described for fast thumb flexion over a small movement distance that agonist bursts have a minimum duration of around 60 ms. Thus, the agonist bursts overlap antagonist bursts with about 30 ms. For larger movement distances (and durations) Marsden et al. (1983) reported silent periods between bursts.

The aim of this paper is to verify whether the triphasic control is time optimal. Then, it will be tried to explain why the third phase is needed. Also, the relevance of differences in timing of agonists and antagonists and the relevance of submaximal amplitudes will be discussed. It will be kept in mind that possibly apart from the movement duration other factors are optimized. Also, certain aspects of the triphasic pattern may reflect the organisation of movement control rather than the control objectives.

## Methods and results

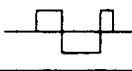

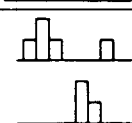
When a "maximal" or "bang-bang" control can be assumed, optimal controls can be derived by maximization of the Hamiltonian (Sim 1988; Pandy et al. 1990; Pandy and Zajac 1991). One of our questions is whether a maximal second and third block are optimal. Therefore, a more general parametric input optimization approach was chosen. Thus, the assessment of the time optimal control requires:

- A dynamic model of the neuromuscular system;
- a definition and parametrization of the input(s) and
- a criterion for the movement duration.

First we will consider a simple linear model developed in Happee (1990, 1991). With this model some concepts will be introduced and the time optimal control will be derived analytically. As a second step we will consider a non-linear model from the literature which can be considered to describe the relevant dynamics of the intact movement system. It will be shown that the criterion for the movement duration used for the linear system cannot be used for this non-linear system. Two alternative criteria will be formulated for which the optimal input will be iteratively determined. Table 1 summarizes the models, input parametrizations and criteria for the movement duration considered.

Formulating these criteria, we will consider the movement duration, from the first input signal received in the muscular system to the time the target is reached. Thus the stimulus reaction time will be disregarded. In our experiments only minor overshoots were found for

**Table 1.** Models, input parametrizations and criteria. (MT = Movement Time;  $x$  = state;  $x_d$  = desired state;  $e$  = position error after  $t = MT$ ;  $MT_e$  = estimated movement time;  $T$  = end of simulation time)

Model	Input parametrization	Criterion; minimize $J$ under constraint $C$	Results, optimal response found with:
Linear, third-order		$J = MT$ $C: x = x_d$	Three maximal blocks
Non-linear, eighth-order		$J = MT$ $C:  e  \leq tol$	Three maximal blocks with synchronous agon/antagonist switching
		$J = \sum_{t=MT_e}^T e^2$	

fast movements (typically 5%). For instance Lehman and Calhoun (1990) describe overshoots up to 50%. We have the impression that large overshoots occur when emphasis is laid on the obtained peak velocity rather than a fast standstill at the desired position. To verify whether an overshoot is functional, we will make no assumptions about an overshoot and we will consider the movement ended only when the target is reached and further maintained.

#### A third-order linear model

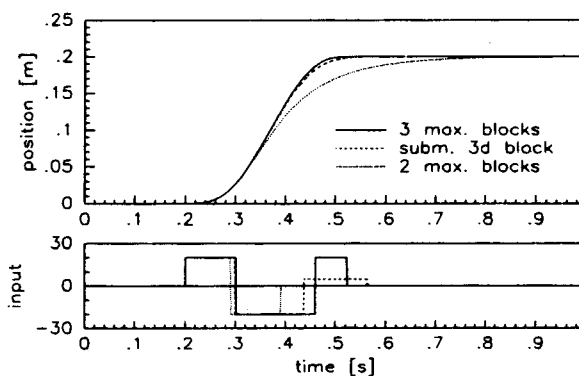
In Happee (1990, 1991) a model is presented with which a muscular input signal can be reconstructed from a movement trace. In this model, agonist and antagonist inputs are combined into one input signal. The hand position follows this input with the following transfer function:

$$H(s) = \frac{1}{T_1 s + 1} \frac{1}{s^2} \quad (1)$$

This simplification can be argued as follows: For agonists and antagonists an identical first-order relation with time constant  $T_1$  is assumed between input and force. So, agonist and antagonist inputs can be combined into one summed input and the states of agonists and antagonists can be combined into one summed state.

For this linear model, Fig. 2 demonstrates that a certain constant position can be reached with an input of two blocks. However, this position will be reached in infinite time due to the time constant  $T_1$  in the filter. As illustrated by the two fastest responses in Fig. 2, with an input of three blocks, the desired state can be reached at the moment the last block switches off; a dead-beat control. The fastest response in Fig. 2 is the time optimal control; a bang-bang control with 2 switches as could be expected for a third-order system (see introduction).

Mathematically, the desired state is reached at the end of the input sequence if at that time the position



**Fig. 2.** Linear model, effect of the third block on movement time, given an equal amplitude of the first two blocks and  $T_1 = 0.1$  s

equals the desired position and the other state variables, velocity and acceleration are zero. Assume that:

$$A = \text{maximal input}, \quad -A = \text{minimal input}; \quad (2a)$$

$$T_a = \text{duration first block } (U = A); \quad (2b)$$

$$T_b = \text{duration second block } (U = -A); \quad (2c)$$

$$T_c = \text{duration third block } (U = A); \quad (2d)$$

$$D = \text{desired displacement}. \quad (2e)$$

Then, integration over the three input blocks and assuming the integrated states equal to the desired states yields for acceleration, velocity and position respectively:

$$0 = A(1 - \exp(-T_a/T_1))(\exp(-(T_b + T_c)/T_1)) - A(1 - \exp(-T_b/T_1))(\exp(-T_c/T_1)) + A(1 - \exp(-T_c/T_1)); \quad (3a)$$

$$0 = A(T_a - T_b + T_c); \quad (3b)$$

$$D = A(\frac{1}{2}T_a^2 + T_a(T_b + T_c) - \frac{1}{2}T_b^2 - T_bT_c + \frac{1}{2}T_c^2). \quad (3c)$$

When the maximal input  $A$  is known, from these equations the durations  $T_a$ ,  $T_b$  and  $T_c$  can be computed. The target is met at the end of  $T_c$  so the movement time  $MT$  equals  $T_a + T_b + T_c$ . On the other hand when  $MT$  is known,  $A$ ,  $T_a$ ,  $T_b$  and  $T_c$  can be computed. Thus,  $T_a$ ,  $T_b$  and  $T_c$  can be expressed as a function of  $T_1$  and  $MT$ . Figure 3 describes that for larger values of  $T_1/MT$  a larger third block is part of the time optimal control.  $T_1$  describes muscular lag, so the desired size of the third block depends on the ratio of muscular lag and movement time. If  $T_1$  approaches zero, the system to be controlled degenerates from a third-order to a second-order system, and only two blocks (with one switch) are required for a time optimal control.

The inputs estimated with this model from experimental movement traces (Happee 1990, 1991) have a third block with a similar duration as the first block, but with a lower amplitude. When assuming such a shape for the input signal, again the third block can be chosen such as to obtain a dead-beat control. Figure 2 shows that this results in a slightly higher movement duration. The magnitude of such a third block is also

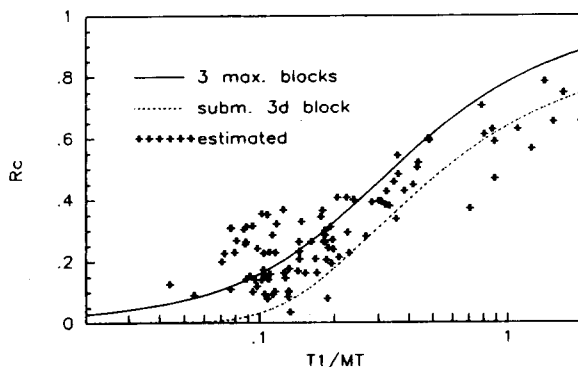


Fig. 3. Linear model, time optimal size,  $R_c$ , of the third block as a function of  $T_1/MT$  which is the neuromuscular time constant divided by the movement time. Lines are analytically derived, points are estimated from maximally fast movements.  $R_c$  is the area of the third block relative to that of the first block

depicted in Fig. 3 together with values estimated with the model from data of maximally fast movements in Happee (1990, 1991).

#### A non-linear model

In the linear model the agonist and antagonist inputs were combined into one summed input and the states of agonists and antagonists were combined into one summed state. Assuming a more realistic non-linear description of muscular dynamics, agonist and antagonist inputs and states can no longer be superimposed. For the linear model the maximum principle implies switching between a maximal agonist and a maximal antagonist input. When defining a separate agonist and antagonist input, maximality only implies switching between zero and maximal inputs. Maximality does not imply that switching of agonists and antagonists should occur simultaneously.

To evaluate the effects of non-linear muscle properties, a suitable model of neuromuscular dynamics was sought. From linear control theory it is known that the order of a system largely determines the number of input switches required for a time optimal control. So, the model should have the order required to describe all relevant dynamical processes which are:

- 1) Recruitment dynamics; representing neural excitation processes (pure time-delays will be discarded since they don't affect the optimal control);
- 2) active state dynamics;
- 3) a force-velocity relationship (Hill curve) together with a series-elastic element and
- 4) limb inertia.

A real muscle input consists of pulse trains for each motor unit and can be modelled as such (Hannaford 1990). Hatze (1981) defined two more global inputs in terms of the relative fraction of active fibres and the average stimulation frequency for these fibres. Most authors define only one neural input which represents muscle force relative to its maximum, assuming stationary isometric conditions (Winters and Stark 1985; Audu and Davy 1985; Zajac 1989).

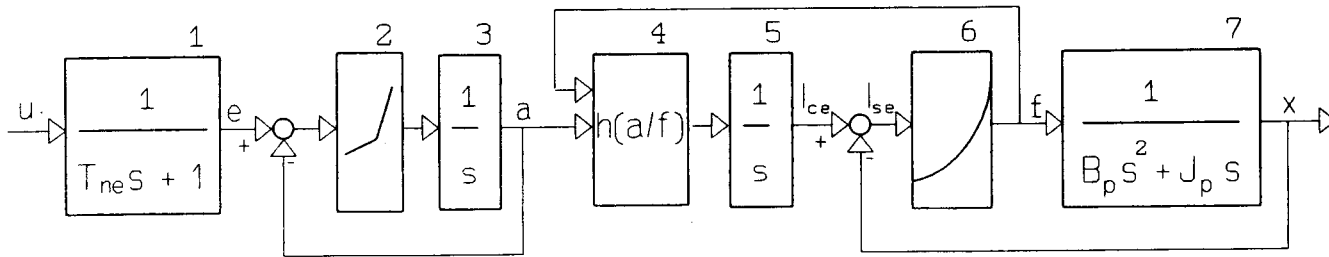
Winters and Stark (1985) modelled the recruitment dynamics as a first-order system. Others do not separately describe recruitment dynamics. Active state dynamics can be modelled in various ways, of which the most essential feature is a fast rise and slow decay. Descriptions of the force velocity relationship are mostly based on the equation formulated by Hill (1938). The series-elastic element is mostly described by an exponential function.

For this contribution, the model of Winters and Stark (1985) was used mainly because it contains all the dynamic processes (1-4) stated above and because it is not unnecessarily complicated. A parallel elastic element is not included because it does not affect the order of the model and because in the movements we studied no extreme joint angles occurred. Our experimental data (Happee and Daanen 1991a, b) concern anteflexion/retroflexion of the humerus. In our group, a detailed dynamical model of the shoulder girdle is currently being developed (Pronk 1991; Veeger et al. 1991; Van der Helm et al. 1991; Van der Helm and Veenbaas 1991). We intend to calculate time optimal inputs for this model in the near future. However, such results will not be generalizable to movements of other joints. Therefore, for the current article, we describe the shoulder as a joint with one degree of freedom; an anteflexion/retroflexion in the sagittal plane. Only two muscles are defined in the model; an anteflexor and a retroflexor. The anteflexor describes the lumped effects of mainly the pectoralis major and the anterior deltoid. The retroflexor describes the lumped effects of mainly the latissimus dorsi and the posterior deltoid. Muscle parameters were described in terms of joint angles and torques instead of muscle length and force. Maximal isometric joint torques were measured in one subject. Limb inertia was computed from cadaver data (Veeger et al. 1991). For the other parameters in Table 2, the values were taken in the range described by Winters and Stark (1985).

This model contains two states for the arm; joint position and velocity. Each muscle has three states; the neural excitation, the active state and the length of the contractile element (Fig. 4). Thus eight states are defined. For the linear model the time optimal input was defined as the input which brings the arm from the present to the desired state in a minimal time. The desired state could be meaningfully defined as the desired position and a zero velocity and acceleration. For the non-linear model the desired state is only partially known; the desired position and velocity are the same as for the linear model. The desired zero acceleration requires that the agonist muscle force equals the antagonist muscle force. Also, the muscular states must be such that over the whole future, agonist and antagonist force will be equal. This, for instance occurs when both the muscles have identical states, parameters and (stationary) inputs. Such identical states would put 3 constraints on the muscular states. Thus, the desired state would be a 3 dimensional subspace of the 8 dimensional state space. To reach this desired subspace, it requires an input with at least 5 free parameters (see introduction)

**Table 2.** Parameters of the system to be controlled

Description	Parameter	Value
Maximum isometric torque	$M_{max}$ [Nm]	100
Limb inertia defined towards joint	$J_p$ [kgm <sup>2</sup> ]	0.25
Joint damping	$B_p$ [Nms/rad]	0.2
SE-stretch with maximal isometric force	$SE_{xm}$ [deg]	30
Series-Elastic shape	$SE_{sh}$ [ ]	3
Hill, maximal shortening velocity	$MV_{vm}$ [rad/s]	25
Hill, shape parameter	$MV_{sh}$ [ ]	0.25
Time-constant, increasing active state	$T_{ac}$ [ms]	10
Time-constant, decreasing active state	$T_{da}$ [ms]	50
Time-constant, neural excitation	$T_{ne}$ [ms]	40



**Fig. 4.** The system to be controlled, only one muscle is shown, parameter values are given in Table 2; block from left to right: 1 neural excitation dynamics; 2, 3 active state dynamics, block 2 has gain  $1/T_{ac}$  for positive inputs and gain  $1/T_{da}$  for negative inputs; 4 force-velocity relation, shortening velocity is a monotonously rising

function ( $h$ ) of active state divided by force; 5 shortening velocity integrated to shortening of contractile element ( $L_{ce}$ ); 6 force is an exponential function of stretch of the series elastic element ( $L_{se}$ ) which results from shortening of the CE and the external displacement ( $x$ ); 7 limb inertia ( $J_p$ ) and joint damping ( $B_p$ )

But it can also be that after the target is reached, non-stationary inputs are required to keep agonist and antagonist muscle force equal. So, for this non-linear system the number of states gives no clear prediction of the time optimal input.

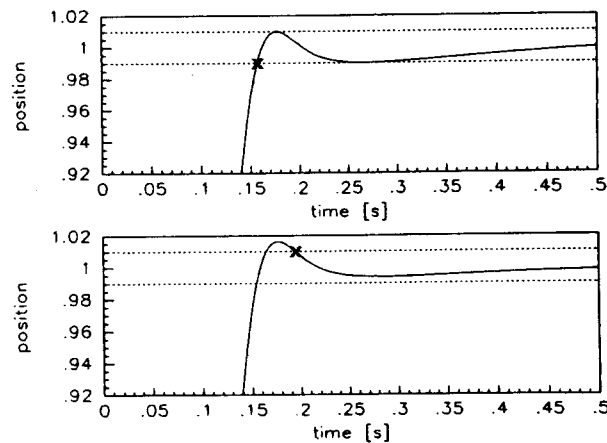
To circumvent the definition of a desired state, only the desired output will be regarded. The position is the only output considered since a correct position over a certain time interval implies a zero velocity and acceleration. Ideally, after the movement duration the position  $x$  should be equal to the desired position  $x_d$ . The position error  $e$  was defined relative to the desired position  $x_d$  and the movement distance  $D$ :

$$e = (x - x_d)/D. \quad (4)$$

As preciseness is not stressed in fast movements, a certain tolerance ( $tol$ ) around the desired position will be allowed. The movement time  $MT$  is thus defined as the interval between the start of the first muscle input and the time after which  $|e| \leq tol$ . Following this definition, as can be seen in Fig. 5, for a movement with an overshoot above the tolerance interval, the movement time is defined as the time when the position re-enters the tolerance interval whereas for a response with no such overshoot, the movement time is the first time at which the tolerance interval is entered. This leads to a discontinuity of  $MT$  as a function of inputs. Therefore, the time optimal control will be sought separately for responses with and without an overshoot above the tolerance interval. Later the preferable of these two solutions will be established.

As described in Table 1, for the non-linear model two kinds of input parametrizations will be considered. The first kind of input parametrization assumes bang-bang inputs of 3 blocks. As it is not yet proven that the time optimal control consists of three blocks, a second parametrization of inputs is implemented which describes discretized input amplitudes.

Blockwise inputs are parametrized by block durations and amplitudes and by durations of silent periods between blocks where also negative silent periods are



**Fig. 5.** Movement time defined as the point (X) after which the position stays within the tolerance interval. An overshoot above the tolerance interval is present only in the lower trace

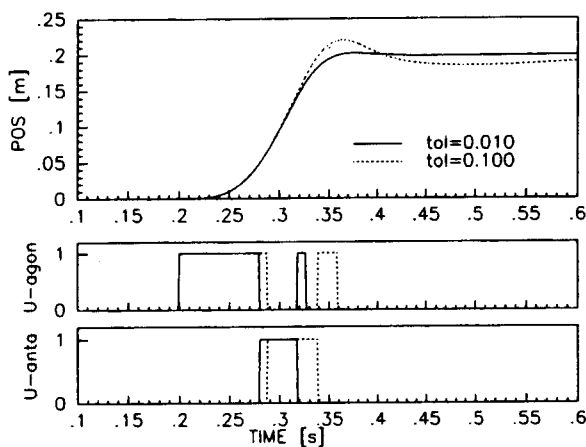


Fig. 6. Time optimal movements with an input of three maximal blocks as a function of "tol" defining the tolerance interval around the desired position

allowed resulting in overlapping blocks. For an input of three blocks this leads to eight parameters to optimize. For optimization both the class of gradient based iterative methods and the class of grid search methods were considered. The major problem with gradient methods is that in the case of non-linear equations no proof can be given that the solution obtained describes the absolute minimum of the criterion to be minimized. A grid search can discriminate between local and global minima, but a minimum somewhere in between the grid points can escape the attention. Also, for most grid points there will exist no time after which  $|e| \leq tol$  and thus no movement time can be defined. Considering this, a dual approach was chosen: Optimal block durations were computed iteratively with a gradient based method (Yin Yu Ye 1987). This iterative minimization was performed for several grid values for amplitudes and silent period durations.

First, three blocks of maximal amplitudes without silent periods were considered. So, three block durations were to be optimized. Several values for the tolerance  $tol$  were implemented. As could be expected, higher overshoots are observed for larger values of  $tol$  (Fig. 6). For values of  $tol$  below 0.01 only minor changes in the controls and the response are observed. For lower values of  $tol$ , the computation of the optimal input is very time-consuming. For values of  $tol$  below 0.003 no reliable results were obtained. Therefore,  $tol = 0.01$  is further considered as a basis for comparison. In the results presented in Fig. 6, no overshoot above the tolerance interval was assumed. When assuming such an overshoot, results invariably converged to a minimal overshoot. From this, it can be concluded that an overshoot above the tolerance interval is never optimal for the described criterion. So from now on, the movement time criterion will be computed with  $tol = 0.01$ , and no overshoot above the tolerance interval will be allowed.

For several grid values of block amplitudes and silent period durations, optimal values were computed for the three block durations (Table 3). For all combi-

nations of block amplitudes and for reasonable silent period durations, a set of block durations could be found which led to a movement duration at most 9% larger than minimal. A minimal movement time was obtained with maximal blocks. An overlap of blocks (negative  $S_2, S_3$ ) seems to give a slight reduction of the movement time but this reduction is negligible given the accuracy of the optimization. Silent periods between blocks (positive  $S_2, S_3$ ) led to an increased movement time. An input matching the experimentally found activation patterns is marked (\*) in Table 3 and simulated in Fig. 7. These results indicate a sub-optimality of the activation patterns found in the EMG. However, Fig. 7 also shows a considerable reduction of torques. This suggests that a criterion is minimized which also takes into account the effort spent.

Next, the optimization of discretized inputs was performed to verify whether, as assumed above, the optimal control consists of three blocks. With the discretized inputs, also the optimality of silent periods and submaximal amplitudes can be assessed once more. The two inputs were discretized for time steps of 0.01 s (the simulation time step is at most 0.001 s). Inputs were considered over 0.20 s which led to 40 parameters to optimize which yielded large computational costs.

The described criterion  $tol = 0.01$  (Fig. 5) could not be met with these discretized inputs due to numerical instability. Therefore, a simpler criterion was formulated as the sum of squares of the position error after a certain time  $MT_e$ , where the time  $MT_e$  is a pre-estimated movement duration. For  $MT_e = 160$  ms with block shaped inputs, the squared criterion yielded practically identical results as the criterion  $tol = 0.01$ .

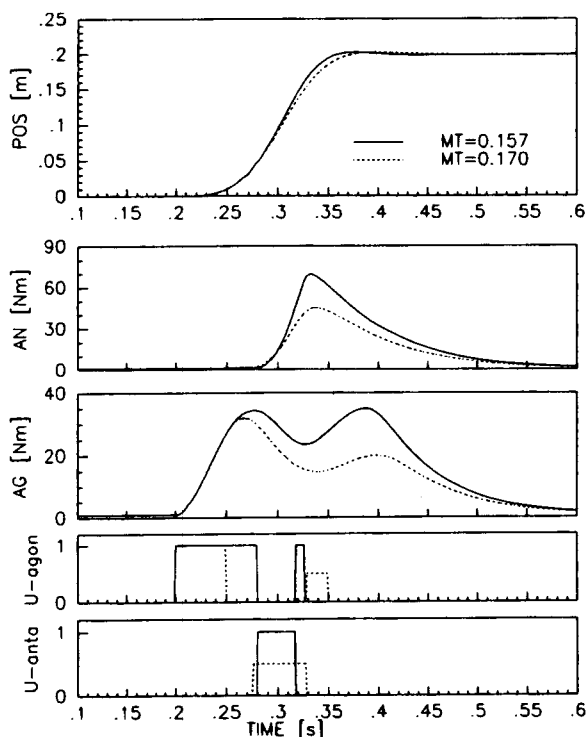
Optimization of the squared criterion for discretized inputs again led to an input of three phases with an almost synchronous agonist/antagonist timing (Fig. 8). After a few iterations a very small fourth phase was estimated but further optimization of a four block input in no case resulted in a shorter movement than obtained with a three block input. So, it is concluded that a three block input is optimal within the accuracy obtained in our computations.

#### Effects of gravity

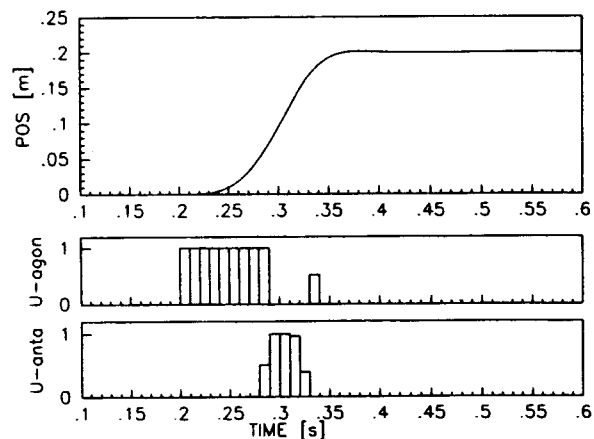
In the simulations described above, the effect of gravity has been neglected. However, in Happee and Daanen (1991a, b) antagonist activity was found in the initial position before movements and agonist activity was found in the final position after movements. Thus, an equilibrium point must lie somewhere between the initial and the final position. This can be attributed to gravity as was shown in a mechanical analysis (Happee and Daanen 1991b). However, passive elasticity of joint and muscles can also be relevant in here. Both the initial antagonist EMG and the final agonist EMG were about 15% of maximal amplitudes as found during the first agonist burst. To assess the effect of gravity on the movements considered, gravity was implemented matching 15% antagonist activity in the initial position and 15% agonist activity in the final position.

**Table 3.** Optimal block durations ( $T$ ) and movement time ( $MT$ ) as a function of block amplitudes ( $A$ ) and silent period durations ( $S$ );  $S_2$  is the silent period between first and second burst,  $S_3$  between second and third burst, negative values indicate overlap.  $S$ ,  $T$  and  $MT$  are given in ms. The input matching experimental EMG is marked (\*)

Fixed					Optimized			$MT$	$MT/MT1$
$A_a$	$A_b$	$A_c$	$S_2$	$S_3$	$T_a$	$T_b$	$T_c$		
1	1	1	0	0	80	38	9	157 = $MT1$	
0.9	1	1	0	0	83	35	8	162	1.035
1	0.9	1	0	0	78	41	8	159	1.018
1	1	0.9	0	0	79	37	8	159	1.014
1	0.5	1	0	0	67	71	10	167	1.067
1	1	0.5	0	0	79	36	13	159	1.016
1	0.5	0.5	0	0	67	69	19	168	1.070
1	1	1	-30	0	?	?	?		
1	1	1	-20	0	99	43	1	156	0.997
1	1	1	-10	0	90	41	6	155	0.992
1	1	1	-5	0	85	39	7	156	0.996
1	1	1	5	0	75	35	9	160	1.019
1	1	1	10	0	71	35	11	158	1.012
1	1	1	20	0	62	32	12	161	1.025
1	1	1	40	0	47	27	14	166	1.058
1	1	1	0	10	79	36	5	160	1.018
1	1	1	0	-10	80	39	11	156	0.997
1	0.5	0.5	25	0	50	54	21	170*	1.088
1	0.5	0.5	10	0	60	63	20	168	1.074
1	0.5	0.5	-10	0	75	76	17	168	1.075
1	0.5	0.5	0	25	67	65	11	171	1.090
1	0.5	0.5	0	10	67	66	14	171	1.090
1	0.5	0.5	0	-10	67	72	22	167	1.065

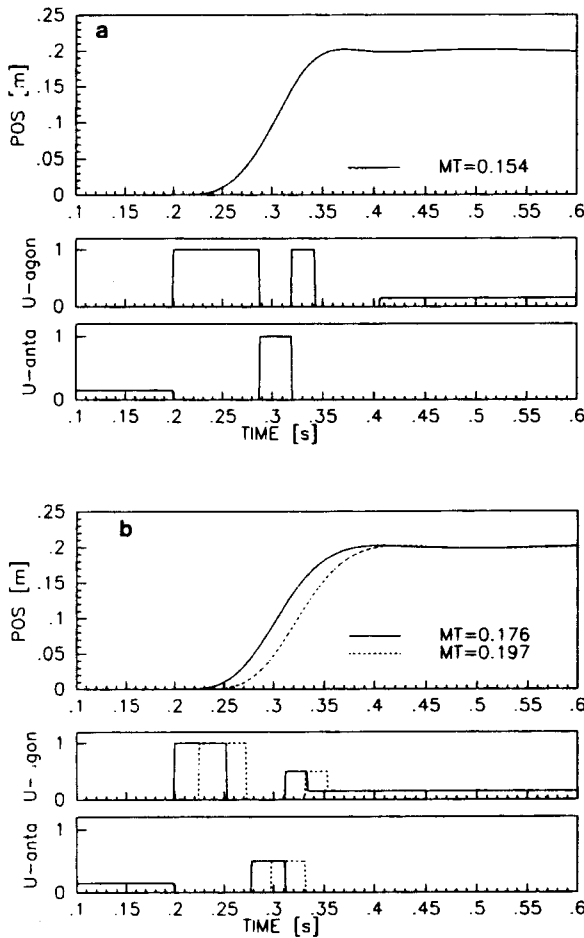


**Fig. 7.** Time optimal control (—) and suboptimal control (...); the suboptimal control matches EMG signals found



**Fig. 8.** Discretized inputs, parameters describe input amplitudes over intervals of 0.01 s

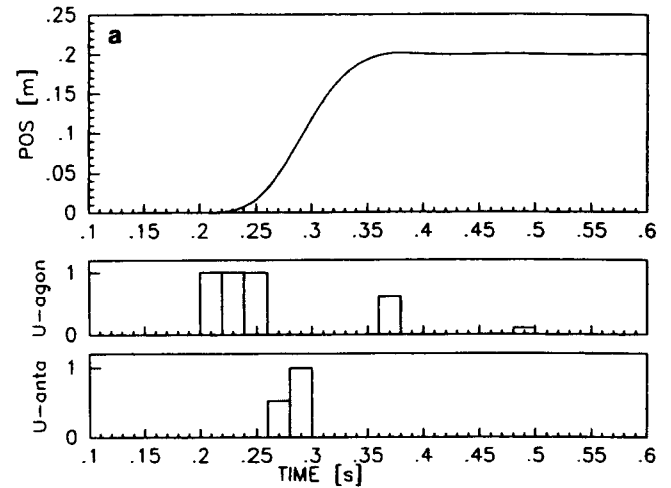
For this model, including the effects of gravity, again an input of three maximal blocks without silent periods was considered. At first it was assumed that right after the third burst, a continuing 15% activity should occur. However, for such an input, the criterion ( $tol = 0.01$ ) could not be met within reasonable time. Therefore, again a discretized input was optimized. This yielded an input of three maximal blocks followed by



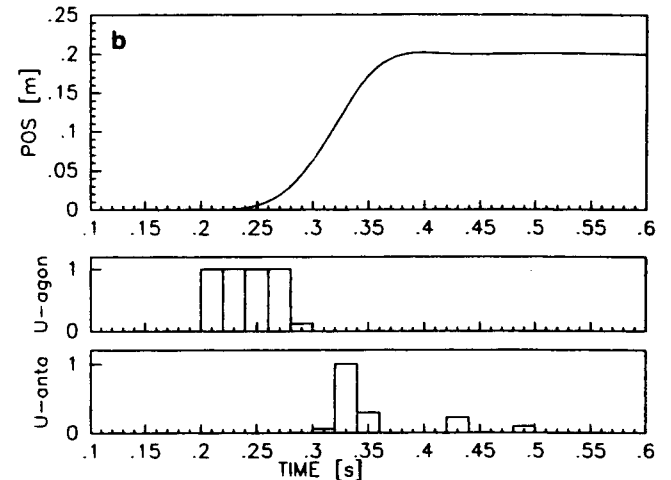
**Fig. 9.** **a** Simulation with gravity, time optimal control. **b** Simulation with gravity, inputs matching EMG signals found but without a silent period before the first agonist burst (—) and inputs fully matching EMG signals found (....)

a silent period before the final 15% agonist activity a started. Optimizing the three block durations and this silent period, yielded the result in Fig. 9a, where a considerable silent period is shown after the third burst. Such a silent period was not found in the EMG. So, the model including gravity reveals another difference between the optimal control and EMG traces.

As said, without such a silent period after the third phase, the criterion  $tol = 0.01$  could not be met with maximal blocks. But, in recorded EMG traces, submaximal amplitudes were observed in the second and the third phase. As can be seen in Fig. 9b for such inputs the criterion  $tol = 0.01$  can be met. So, it can be concluded that both the submaximal amplitudes and the silent period between agonist and antagonist burst contribute to reaching the target without a silent period after the third phase. One of the two responses in Fig. 9b contains a silent period between the end of a pre-movement antagonist activity and the agonist burst as was also found in the EMG. Apparently this silent period just delays the response and seems to have no particular advantage.



**a)** fast agonist, slow antagonist



**b)** slow agonist, fast antagonist

**Fig. 10a, b.** Different dynamics for agonist and antagonist muscle; fast means that all time-constants are reduced to 80% of values in Table 2, slow means an increase of time-constants to 120%

#### *Differences between agonist and antagonist dynamics*

In all previous results, identical parameters were used for agonist and antagonist muscles. Therefore, a similarly fast decay of force occurs in agonist and antagonist muscles. Hereafter it will be assessed to what extent a difference in dynamics affects the optimal control. In Fig. 10 results are shown for either a faster agonist with a slower antagonist and vice versa. These results were obtained by optimization of discretized inputs. Both the responses in Fig. 10 contain silent periods, and some activity after the target is reached. With a simpler input of two or three blocks, satisfactory responses could not be obtained.



## Discussion

Both the linear model with one input and the non-linear model with two inputs were used to evaluate the time optimal control for goal directed movements.

Because the linear model is of a third-order it adequately predicts the time optimality of a triphasic control. This model is of a third order because apart from a second order limb dynamics, a first order neuromuscular dynamics is included. The computed time optimal size of the third block depends on the ratio of the neuromuscular time constant and the movement time (Fig. 3). This may explain dependencies of the occurrence of a third phase on the joint system to be controlled.

For the non-linear model it was found that the time optimal input, as expected, switches between maximal agonist and maximal antagonist activation. This indicates a sub-optimality of the activation patterns found in the EMG. However, an input matching the experimentally found activation patterns led to an increase in the movement duration with only 9% and to a considerable reduction of muscle forces (Fig. 7). This suggests that a criterion is minimized which also takes into account the effort spent. Another point is the organisation of movement control. A pure open-loop control may well be a maximal control; such a control just requires the optimal switch times to be known. A feed-back control can also be maximal; such a control requires a criterion for input switches as a function of proprioceptive information eventually combined with information from an internal representation. Such a control implies a modification of input timing rather than amplitude. However, the submaximal amplitudes we found in the second and third phase also allow amplitude modification as could be obtained by proportional feedback. Gottlieb et al. (1990) relate EMG amplitudes and acceleration to instructions regarding movement velocity and not to the desired displacement. This suggests a switching of inputs but now to task dependent levels. These levels may actually be determined by the size of the recruited motoneuron populations. Thus, a possible explanation for the submaximal second and third phase is that motoneuron populations allocated to each input phase have different sizes.

Including gravity in the non-linear model again led to the optimality of an input of three maximal blocks now followed by a silent period before a final agonist activity starts (Fig. 9a). An input without this silent period, and with three maximal blocks led to an inaccurate movement. An input matching EMG signals found, again led to accurate movements (Fig. 9b) where it can also be seen that a silent period before the first agonist burst as found in the EMG merely led to a delayed movement and thus apparently does not serve any movement objective. (In our experiments, a fast movement or a short movement duration was stressed rather than a short reaction time.) The silent period can reflect aspects of the organisation of movement control; it can reflect the so called motor preparation where Manning and Hammond (1990) found modified reflex gains before the onset of voluntary EMG. Mellah et al.

(1990) found activity in low-threshold motor units after a warning signal but before the actual stimulus.

With different dynamics of agonist and antagonist muscles, considerable silent periods were estimated between the first and second or the second and the third burst (Fig. 10). This yields the important conclusion that the optimality of synchronous switching of agonist and antagonist as found before is only due to the identical dynamics of agonist and antagonist. Thus optimality of synchronous switching cannot be assumed generally. Here it is interesting to note that Brown and Cooke (1990) show in a special movement paradigm that the relative timing of agonist and antagonist need not at all be synchronous or even fixed.

The small activity in Fig. 10 after the target is attained indicates that in this particular condition further inputs are required to achieve a cancellation of the decaying agonist/antagonist forces.

In all cases considered, the optimal control contained maximal amplitudes for the first three blocks. This contradicts the submaximal amplitudes as supposed optimal by Hannaford and Stark (1985, 1987).

In Fig. 7 it can be seen that after reaching the desired position, muscle forces decrease rather slowly but agonist and antagonist force are practically equal. This illustrates that for this system, a stationary output (position) does not imply stationary states. This is why a unique desired state does not exist for this system which complicated the formulation of a criterion to optimize.

This also illustrates the role of the third phase. Shortly after the third phase, agonist and antagonist forces become practically equal which provides an equilibrium in the desired position. From computations not presented here it emerged that without a third phase an equilibrium was reached much later. Only if due to a difference in dynamics, the antagonist force decays faster than the agonist force, no third agonist phase is required (Fig. 10b). From this it is concluded that the third phase serves to compensate the slowly decaying antagonist force.

In the simulated non-linear model, neural excitation and active state dynamics are described by two first-order systems in series (Fig. 4). In the literature these dynamics are often described together by one first-order system. We also evaluated the effect of such a reduction of the model-order. The time-constant  $T_{ne}$  was chosen zero.  $T_{ac}$  and  $T_{da}$  were chosen such that for the optimal input in Fig. 8 the active state of the simplified model best matched the active state with the original model. For the simplified model the optimal response was very close to the original response but the third phase was estimated to be much shorter. This indicates that the optimal size of the third block is not only dependent on a global measure for muscle dynamics as was indicated by results of the linear model but also on the precise formulation of muscle dynamics.

A question yet to be answered is in how far the current results can be generalized to multijoint movement. Interaction forces may seriously complicate matters. Maximality of inputs is not expected to occur in all joints (or muscles) as it is unlikely that the available forces are fully used in all joints. In most movements, only the available forces in one joint will be rate limiting.

### Numerical aspects

It was found that strongly different inputs led to almost identical responses. This can be explained by the fact that the movement system acts as a low pass filter. In particular, an eventual overlap of the first two blocks or a silent period between these blocks had very small effects on the movement and its duration. The same applies for the amplitudes of the second and third block (Table 3). Here it must be stressed that the effect of a change of the "fixed" parameters in Table 3 is only small because for each row, the other parameters were optimized given these fixed parameters.

The three block pattern is described by eight parameters; three durations, three amplitudes and two silent period durations between blocks. Using gradient based methods for more than three parameters at a time, the numerical conditioning became very poor. This poor conditioning of course arises from the above-mentioned insensitivity of the movement duration to certain changes in the inputs. For this reason, a gradient based optimization was combined with a grid search (Table 3).

For the discretized inputs the conditioning is even poorer as up to 40 parameters are estimated. However, after a few iterations, many parameters obtained minimal or maximal values and therefore these parameters no longer affected the conditioning. Here it must be stated that the results in Fig. 8 were obtained after a very large number of iterations. After a few iterations an almost optimal response was obtained with inputs which were only roughly block shaped.

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