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Dynamic Interactions Between Limb Segments During Planar Arm Movement

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Abstract. Movement of multiple segment limbs requires generation of appropriate joint torques which include terms arising from dynamic interactions among the moving segments as well as from such external forces as gravity. The interaction torques, arising from inertial, centripetal, and Coriolis forces, are not present for single joint movements. The significance of the individual interaction forces during reaching movements in a horizontal plane involving only the shoulder and elbow joints has been assessed for different movement paths and movement speeds. Trajectory formation strategies which simplify the dynamics computation are presented.

1. Introduction

In the movement of a multiple link mechanical structure such as the arm/forearm system, the torques at the joints arise not only from muscles acting at the joints but also from interactions due to movement of other links. These interaction torques are not present during movement at only a single joint and represent a significant complicating factor in the dynamic analysis of the movement. The main purpose of this paper is an assessment of the significance of the interaction torques during two joint arm movement involving the shoulder and elbow joints, and secondarily a consideration of plausible mechanisms for dynamics computation or compensation by the human motor system.

While a dynamic analysis of arm movement reveals the theoretical existence of interaction torques, it is not a priori necessary that these interaction torques are important during normal arm movement. During sufficiently slow movement, for example, the effect of gravity will completely dominate all other dynamic terms. In the field of robotics, moreover, it is usually assumed that the *inertial torques*, which are the torques proportional to joint acceleration, are usually much more significant than the *velocity torques*, which are the torques proportional to the product of joint velocities of two joints. Thus in robotics a considerable simplification of the dynamic analysis is achieved by excluding the velocity torque terms. Finally, in some situations friction can dominate the interaction torques as well, for example movement through a viscous medium like water or the presence of significant sliding friction at the joints of some manipulators.

If the interaction torques are significant during normal movement, then they will influence the trajectory of the arm. In order to control the path that the arm takes during movement, such as a straight line movement with the hand, it would then be necessary to generate the appropriate torques for the movement which include contributions for the interaction terms. Possible sources for the torque generation include preprogramming, feedback correction through loops including sensors, and intrinsic muscle stiffness.

The experimental evidence presented here indicates that the interaction torques are significant for a two joint arm movement over a range of movement speeds and of movement paths. In addition, the significance of the velocity interaction torques relative to the inertial interaction torques does not vary with speed of movement. A fundamental time scaling property of the dynamics has been identified as the reason for this invariance, and is suggestive of a simplifying strategy for dynamics compensation. It is concluded that there must exist control strategies which compensate for the presence of interaction torques during multiple joint movement.

1.1. Paths, Trajectories, and Dynamics

Before proceeding with a discussion of the dynamics of arm movement, it is important to distinguish path

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from trajectory. By a *path* will be meant the three-dimensional space curve that some point on the hand follows; by a *trajectory* will be meant the time sequence of movement along a path.

In planning an arm movement, there is a question as to whether the human motor system plans in the Cartesian space of the hand or in the joint space of the arm. If movements are planned in joint space, then a time sequence of joint angles is found which takes the hand from the starting position to the final position. The main feature of joint space planning is that there is no explicit control of the hand trajectory between the two positions due to the complicated relation between joint angles and Cartesian hand positions. If movements are planned in Cartesian space, then there must be an inverse kinematics computation which converts time sequences of the hand positions to time sequences of the joint positions. A third alternative, such as in final position control (e.g. Sakitt, 1979) where there is no explicit trajectory control, cannot be considered at this time because such strategies have not been resolved for two-joint arm movement.

Because the goal of arm movement is often the placement of the hand, for example in pointing, reaching, or in transporting a grasped object, it can be argued that arm movement planning occurs in the hand Cartesian space. Movements of the upper arm and forearm would then be subservient to the goals of the hand movement and not be subject to explicit planning themselves. In corroboration of this hypothesis, it has been observed that human arm movement, in a task which involves pointing or reaching, results in a straight line path of the hand (Morasso, 1981; Abend et al., 1981; Soechting and Lacquaniti, 1981). This observation has been substantiated by movements measured here, and is consistent with the notion that the path of the hand is the important consideration when arm movements are planned. A straight line movement of the hand is the shortest distance for the hand to move between two points, and it also minimizes the inertial forces on a grasped object (which for example is important to avoid spilling a glass of wine).

Whether the movement of the hand or a time sequence of joint angles directly is being planned, an adequate job of computing or compensating for the dynamic interaction between limbs is required to prevent deviation of the arm from the trajectory. The character and the degree of dynamic interactions is a complex function of the path and of the trajectory.

In order to proceed with a discussion of how dynamic interactions vary with path, a simplified kinematic and dynamic analysis of arm movement will be presented. In this paper we consider arm movements involving only shoulder and elbow joint flexion and confined to a horizontal plane. A simplified model of the kinematic linkage of the human arm is presented in Fig. 1, which actually is a very good approximation to the biomechanics. However, in the computer simulation discussed in the Methods section a more accurate general model is used. During arm movement the hand describes some space curve, and the relation between the joint angles θ_1, θ_2 and the position x, y of the hand is given by:

$$\theta_2 = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right), \tag{1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta_2}{l_1+l_2\cos\theta_2}\right). \tag{2}$$

Dynamics relates a trajectory description in terms of joint angles, rates, and accelerations to the joint torques which realize the trajectory. There are two complementary aspects to a dynamic analysis. The *integral dynamics* yields a trajectory given a time sequence of torque inputs to the joints, while the *inverse dynamics* yields the required torques given a trajectory description. Given a trajectory plan, it is therefore the inverse dynamics problem which must be solved in order to arrive at the joint torques. The dynamic equations to yield expressions for the elbow and shoulder torques n_2 and n_1 can be found from a straightforward application of kinematics, the Newton-Euler equations, and d'Alembert's Principle (Luh et al., 1980):

$$\begin{aligned} n_2 = {} & \ddot{\theta}_1\left(I_2 + \frac{m_2 l_1 l_2}{2}\cos\theta_2 + \frac{m_2 l_2^2}{4}\right) \\ & + \ddot{\theta}_2\left(I_2 + \frac{m_2 l_2^2}{4}\right) + \frac{m_2 l_1 l_2}{2}\dot{\theta}_1^2 \sin\theta_2, \end{aligned} \tag{3}$$

$$\begin{aligned} n_1 = {} & \ddot{\theta}_1\left(I_1 + I_2 + m_2 l_1 l_2 \cos\theta_2 + \frac{m_1 l_1^2 + m_2 l_2^2}{4} + m_2 l_1^2\right) \\ & + \ddot{\theta}_2\left(I_2 + \frac{m_2 l_2^2}{4} + \frac{m_2 l_1 l_2}{2}\cos\theta_2\right) \\ & - \frac{m_2 l_1 l_2}{2}\dot{\theta}_2^2 \sin\theta_2 - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2, \end{aligned} \tag{4}$$

where m_1, m_2 are the masses and I_1, I_2 are the rotary inertias of links 1 and 2 respectively. The inertial torques, which are proportional to the angle accelerations, are comprised of both the normal inertial terms which represent a single joint movement and an interaction term due to movement of another link. For the elbow torque expression (3), for example, the inertial torque proportional to $\ddot{\theta}_2$ is the normal inertial term which would arise in a single joint elbow movement, while the inertial torque proportional to $\ddot{\theta}_1$ is an interaction due to the shoulder movement.

There are two types of velocity torques, both of which represent interactions. If the velocities in the product pair represent two different joints, the terms proportional to this product are called *Coriolis torques.* In the shoulder torque expression (4) the Coriolis torque is proportional to $\dot{\theta}_1\dot{\theta}_2$, but there is no Coriolis torque acting on the elbow joint. When the velocities in the product pair represent the same joint, the terms proportional to this product are called *centripetal torques.* In the elbow torque expression (3), for example, there is a centripetal torque due to the shoulder movement and proportional to $\dot{\theta}_1^2$.

From a dynamic standpoint, planar arm movements can be broadly classified into whipping and reaching actions. A *whipping* action involves flexion of the elbow and shoulder joints in the same direction, while a *reaching* action involves flexion of the elbow and shoulder joints in opposite directions. Some motions are composite, but for most of the straight line paths measured in the course of this research this categorization holds.

During whipping movements the joint velocities of the shoulder and elbow joints have the same sign at the same point in the trajectory, and the joint accelerations have the same sign as well. In terms of the torque at the shoulder joint (4), the inertial torques provide an additional acceleration to the upper arm in the direction in which it is moving. The effect of the velocity torques, which work together because they have the same sign, depends on the acceleration state. For a negative acceleration the velocity torques aid the upper arm movement, for a positive acceleration the velocity torques oppose it.

During reaching movements the joint velocities and joint accelerations of the shoulder and elbow have opposite signs. In the shoulder torque expression (4) the inertial torque from the elbow opposes the movement of the upper arm, while the velocity torques have opposite signs and have a diminished influence as compared to whipping movements. For the particular case of a straight line reaching movement which intersects the shoulder joint, the dynamics of the shoulder take on a particularly simple form because the velocity terms precisely cancel out. Since the upper arm and the forearm/hand combination in humans are nearly of equal length, the equation of a straight line through the origin is $y=mx$ where m is the slope. Substituting into (1) and (2) one finds that $\theta_1=\tan^{-1}m-\theta_2/2$ and after differentiation $\dot{\theta}_1=-\dot{\theta}_2/2$. From the latter relation and (4) it is clear that the Coriolis and centripetal torques at the shoulder precisely cancel each other out. The inertial torques take on a simpler form as well since $\ddot{\theta}_1=-\ddot{\theta}_2/2$, which means that the shoulder torque can be computed merely from the time dependence of one of the angle accelerations:

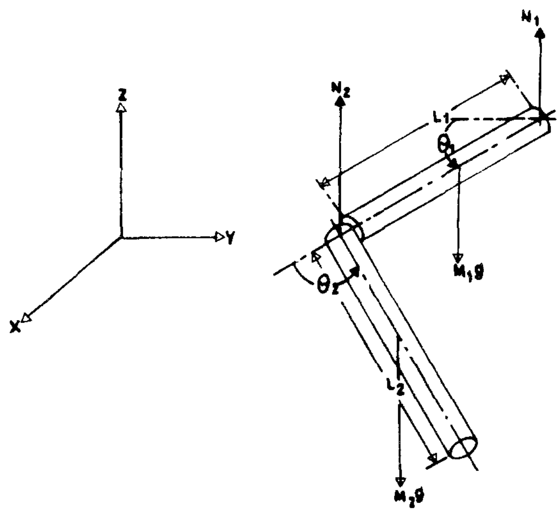

Fig. 1. A simplified model of the human arm involving two links with rotary joints

$$n_1=\ddot{\theta}_1\left(I_1-I_2+\frac{m_1l_1^2-m_2l_2^2}{4}+m_2l_1^2\right).$$

Interestingly the shoulder torque has no dependence on the elbow angle θ_2. For this particular trajectory, therefore, the shoulder joint can behave as if this were a single joint movement. The elbow joint torque however does not simplify as much.

This discussion has illustrated how the dynamics changes with path and trajectory. The added complexity of two joint movement over single joint movement is apparent. If the interaction torques are a significant factor during arm movement, a complex burden will be imposed on the motor system in order to control the movement.

2. Methods

In order to assess the significance of the interaction torques, measurements of human arm movement in a horizontal plane involving the shoulder and elbow joints were made. The resultant kinematic data on the time sequence of joint angles, joint velocities, and joint accelerations were converted into a time sequence of joint torques by solving the inverse dynamics problem. A general purpose simulation program for arbitrary open loop kinematic chains, which can solve both the inverse and integral dynamics, was developed for this purpose. In addition special subroutines were written to compute the magnitude of the various interaction terms contributing to the joint torques. Segmental parameters required in the simulation, such as principal inertias, lengths, masses, and internal axes, were

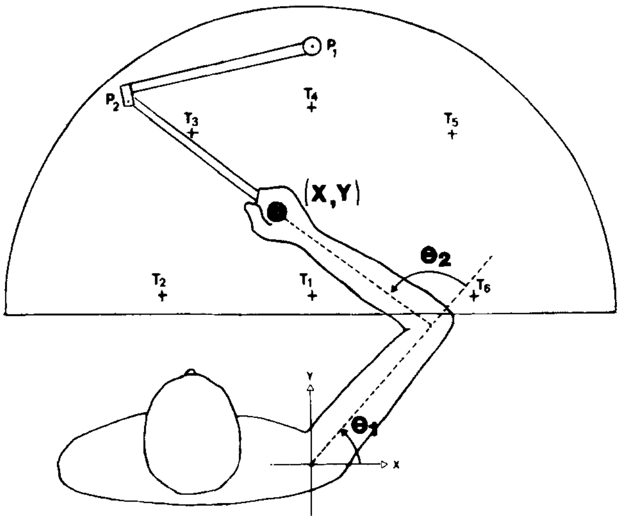

Fig. 2. Experimental apparatus for measuring arm trajectories in a horizontal plane

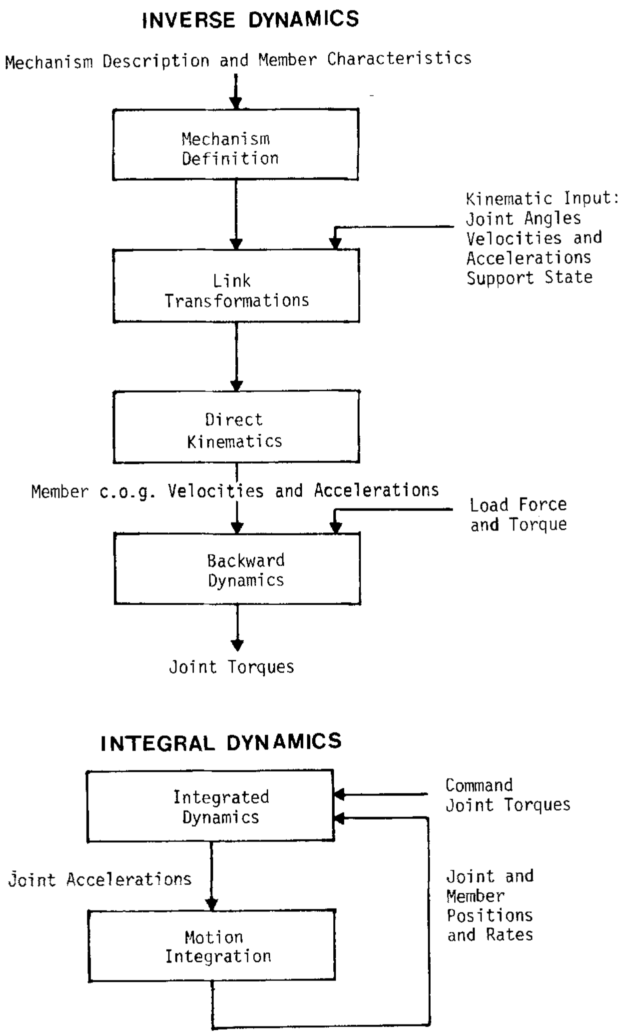

Fig. 3. Block diagram of the simulator for open loop kinematic chains

obtained from a computational model by (Hatze, 1979).

In order to investigate the effect of the various interaction terms on the nature of the trajectories, simulated trajectories were obtained by solving the integral dynamics problem. Command torques were obtained from the joint torques calculated from a measured trajectory by subtracting either the velocity terms or all the interaction terms. By so doing we were able to predict what the trajectories would have looked like were there no provision for the interaction terms.

2.1. Measurement Apparatus

Arm movements were measured with a pantograph gripped by a seated subject and moved in a horizontal plane between specified targets (Fig. 2). Shoulder movement of subjects was restrained by means of straps to the back of the chair, and the wrist was immobilized by bracing. The position of the pantograph handle was determined from potentiometers located at the joints of the mechanical linkage. From a knowledge of a subject's arm and forearm lengths and the location of the subject's shoulder relative to the pantograph, the elbow and shoulder joint angles given the hand position were found from (1) and (2). Velocities and accelerations were obtained by the Lagrange polynomial differentiation method. This apparatus is described in more detail in (Abend et al., 1981).

Six LED targets, which are numbered for reference purposes consecutively from 1 to 6 in the clockwise direction starting from directly in front of the shoulder (Fig. 2) were mounted on a plexiglass cover just above the apparatus. Subjects were asked to move their arms between pairs of targets at various speeds, specified by movement durations ranging from about 0.4 to 1.0 s. Desired time of movement was indicated to a subject by the duration during which the target LED was on. No explicit instructions were given regarding the type of path between targets. Two subjects were tested.

2.2. Determination of Segmental Parameters

According to the method developed in (Hatze, 1979), the human body is divided into 17 segments, and the shoulders are modeled as separate entities. The method subdivides segments into small mass elements of various geometrical shapes thus allowing for the shape and density fluctuations of segments to be modeled in detail. In general no assumptions are made on segmental symmetry. Furthermore the model differentiates between male and female subjects and accounts for the specific body characteristics of each subject. The computations of body parameters are based on a battery of

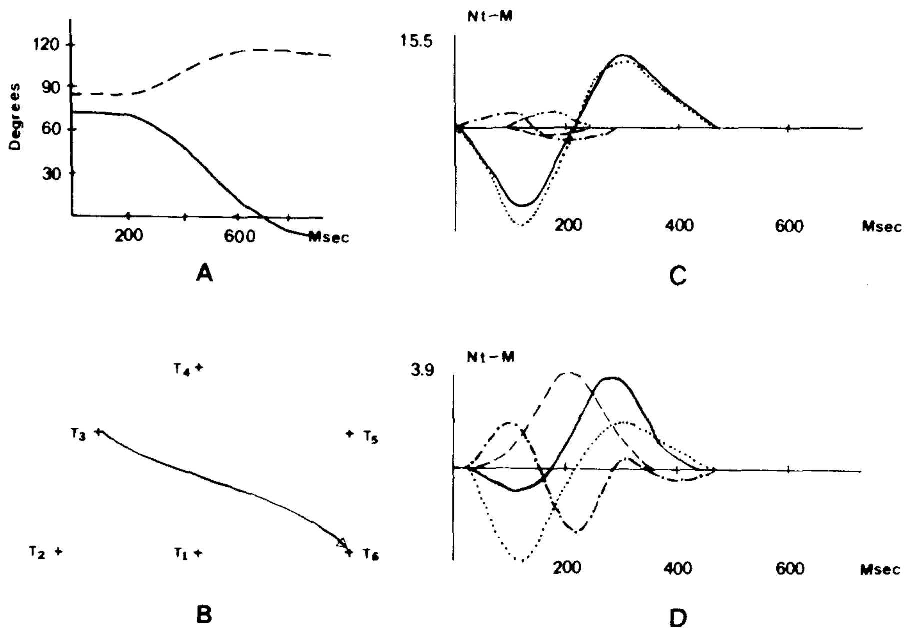

Fig. 4. **A** Joint angle plots for the elbow (dashed lines) and shoulder (solid lines) joints for a 0.5 s movement from target 3 to target 6. **B** A plot of the hand path between the targets. **C** and **D** Components of joint interaction torques at the shoulder **C** and at the elbow **D**. Solid line: the net torque at the shoulder in **C** and at the elbow in **D**. Dotted line: the shoulder inertial torque. Alternating dots and dashes: the elbow inertial torque. Dashes: the centripetal torque due to the elbow in **C** and due to the shoulder in **D**. Two dots with a dash: the Coriolis torque at the shoulder in **C**

anthropomorphic measurements taken directly from the subjects. These include the varying lengths and circumferences of the arm segments, widths of segments, etc.

2.3. The Simulator

The simulator, diagrammed in Fig. 3, has two complementary parts. The first part allows computation of the inverse dynamics, based on an efficient formalism using the recursive Newton-Euler method (Luh et al., 1980). The second part allows computation of the integral dynamics, based on a formalism adapted from (Armstrong, 1979). The latter part allows one to experiment with movement strategies by examining trajectories generated by some hypothetical strategy.

The first module of the simulator sets up the mechanism definition. This includes (1) geometric properties such as the number of chains, the number of links in each chain, internal link coordinate systems, relative movement axes between the links, and internal link lengths, and (2) the inertial properties of mass, location of the center of gravity, and principal inertias for each link. Three coordinate systems associated with each link are defined following the convention suggested by (Orin et al., 1979).

The next two modules of the simulator involve a kinematic analysis of the motion. From the link coordinate definitions and a knowledge of joint angles, the transformation matrices between link coordinate systems and between a given link coordinate system and the reference base can be computed. The angular and linear velocities and accelerations can then be computed recursively from the base to the most distal link. The movement of the base must be specified as the initial conditions for this computation.

The backwards dynamics module computes the joint torques and forces recursively from the most distal member towards the base. The initial conditions required for this computation are any forces or torques exerted by the environment on the chain tip. For free arm movement these external forces and torques are zero.

The integral dynamics modules are shown in the lower part of the block diagram. The integral dynamics solution proceeds by formulating a linear relation between the link linear and angular accelerations and also a linear relation between the link proximal joint forces and the linear accelerations (Armstrong, 1979; Walker et al., 1981). The coefficients of the linear relations are computed recursively distally to proximally. After specifying the initial positions and velocities of all links, the joint accelerations are computed from torques applied at the joints. Integrating by means of a truncated Taylor expansion, the velocities and positions of links at the next time are obtained.

In this paper, the gravity torque has been excluded from the joint torques in computing the dynamics because the movements under consideration occur in a horizontal plane. The elbow joint lies in the same horizontal plane as the glenohumeral joint of the shoulder. Since the elbow is a single degree of freedom hinge joint and the axis of rotation is parallel to the gravity vector, then gravity exerts no torque on the elbow joint. On the other hand the shoulder is a three degree of freedom joint, but once again gravity exerts no torque at the shoulder that affects movement in the

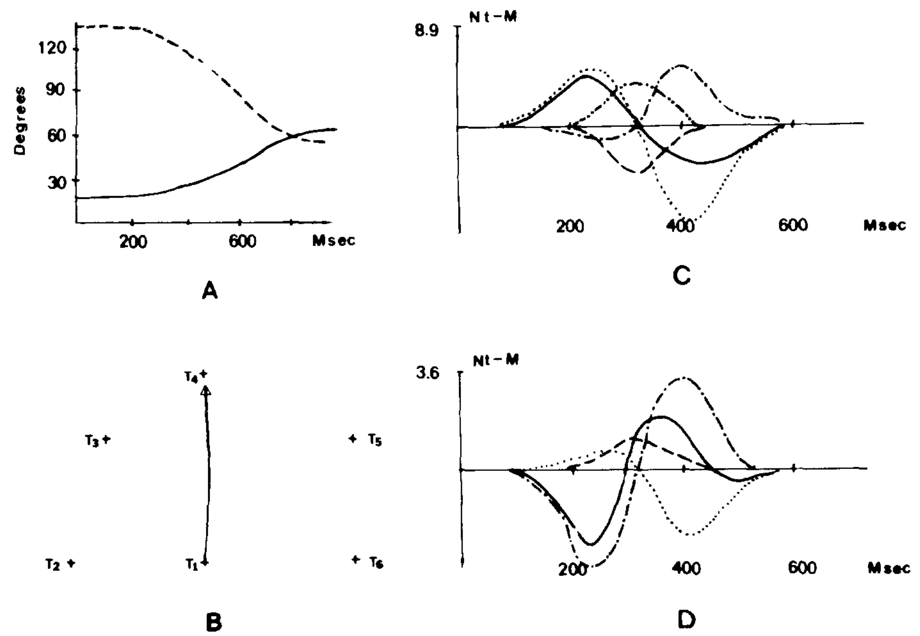

Fig. 5. Plots of joint angles, hand path, and interaction torques for a 0.5 s movement from target 1 to target 4. See the previous figure legend for the description

horizontal plane. Rather, the gravity torque at the shoulder affects the postural control required to elevate and maintain the arm in the horizontal plane. Estimates of the gravity torque will be considered in the context of the magnitudes of the interaction torques for comparison purposes.

There are no explicit terms for viscosity and elasticity due to tendons, connective tissue, and muscles in the torque expressions. The elbow and shoulder joint torques calculated by the simulator include any viscoelastic effects. If one wished to determine the actual forces experienced by the muscles, then a suitable model of the viscoelastic effects must be included in order to partition the joint torques into components due to muscle force and to viscoelastic force. Such considerations however are outside the scope of the present study.

3. Results

3.1. Interaction Torques During Reaching and Whipping Movements

A representative 0.5 s movement from target 3 to target 6 is shown in Fig. 4. The plot of joint angles in Fig. 4a indicates that the joint angles change in the opposite direction, so that this movement may be classified as a reaching movement. A plot of the hand path in Fig. 4b indicates a straight line movement, an observation consistent for this and other measured movements. The components of the joint torques are plotted for the elbow and for the shoulder in Fig. 4c–d. In Fig. 4d the elbow torque plot shows the most complexity in the interaction terms. The inertial and centripetal torques at the elbow due to the shoulder represent a significant fraction of the net elbow torque. The shoulder torque plot in Fig. 4c indicates a small contribution due to interaction terms from the elbow. The centripetal torque is almost zero, whereas the elbow inertial torque and the Coriolis torque individually reach 20% of the maximum net shoulder torque. For this particular movement, therefore, the torque profile of the shoulder is close to that for a single joint movement, whereas the elbow torque profile is quite complicated.

The torque profiles for a representative movement from target 1 to target 4, which is also a reaching movement, are shown in Fig. 5. Here the Coriolis and centripetal torques at the shoulder cancel each other out, as was predicted for straight line paths through the shoulder. On the other hand the elbow inertial torque at the shoulder represents a substantial fraction of the net shoulder torque as opposed to the case for the movements between targets 3 and 6. The centripetal torque at the elbow is somewhat smaller, representing about 30% of the maximum net elbow torque.

The torque profiles for a representative movement from target 2 to target 5 are shown in Fig. 6. The joint angle plot in (A) indicates that the shoulder and elbow joints rotate in the same direction, which means that this movement is a whipping movement. As was predicted for such movements in the introductory section, the velocity terms for the shoulder reinforce each other. The inertial torque profiles however do not clearly indicate a reinforcing effect because of a delay in the onset of elbow movement relative to shoulder movement.

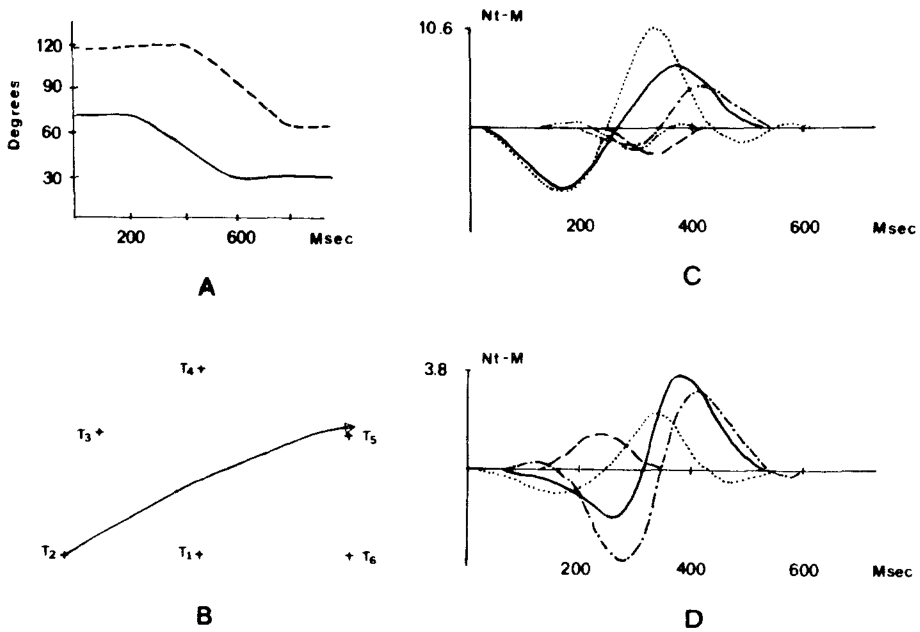

Fig. 6. Plots of joint angles, hand path, and interaction torques for a 0.5 s movement from target 2 to target 5

3.2. *Dependence of Interaction Torques on Speed*

To ascertain how the various interaction terms depend on the speed of movement, a 1.0 s movement between targets 3 and 6 is presented in Fig. 7. The shapes of the torque profiles in Fig. 7 are similar to those in Fig. 4, which represents a movement along the same path but at twice the speed. This result is somewhat surprising in view of the common perception that the velocity interaction terms become significant only at faster movement speeds. In point of fact the velocity terms seem to have the same significance relative to the inertial terms regardless of the speed of movement. The implication of this observation is developed in the discussion.

At the point of greatest velocity the acceleration passes through zero to deceleration. This means that the joint torques at the approximate movement midpoint are dominated by the centripetal and Coriolis interaction terms, because the velocity terms are greatest when the inertial terms are zero. This effect is most severe at the elbow, because of the generally large shoulder centripetal torque component. Therefore, not only does the relative contribution of velocity terms to the net torque remain the same for different movement speeds, but the velocity terms have their greatest effect when the other interaction terms are near zero.

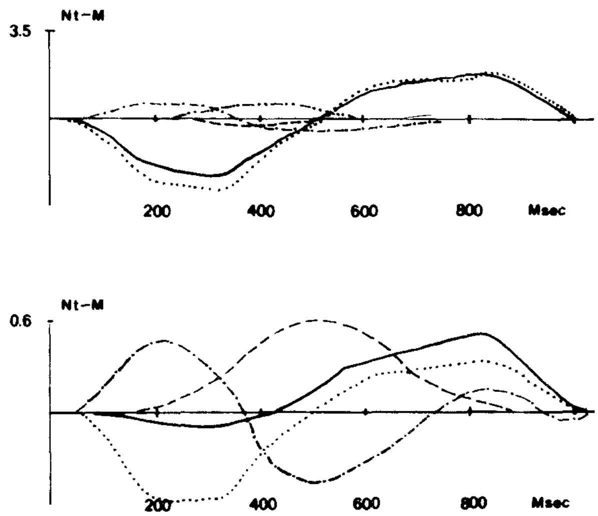

Fig. 7. Components of joint interaction torques for a 1.0 s movement from target 3 target 6

3.3. *The Gravity Contribution*

The gravity contribution averages about 8 newtonmeters at the shoulder, which is greater than the net shoulder torque for the slow movement but less than the net shoulder torque for the fast movements. This is a worst case estimate for the relative effects of gravity and of the interaction forces because of the movement in the horizontal plane. During most human arm movements the gravity contribution will represent a substantially smaller fraction of the shoulder torque. Nevertheless it is clear that even for movements in a horizontal plane the interaction forces are significant relative to gravity and for the faster movements significantly dominate gravity.

3.4. *Simulated Trajectories Without Interaction Terms*

As mentioned earlier subjects make predominantly straight line movements between targets. To get a feel

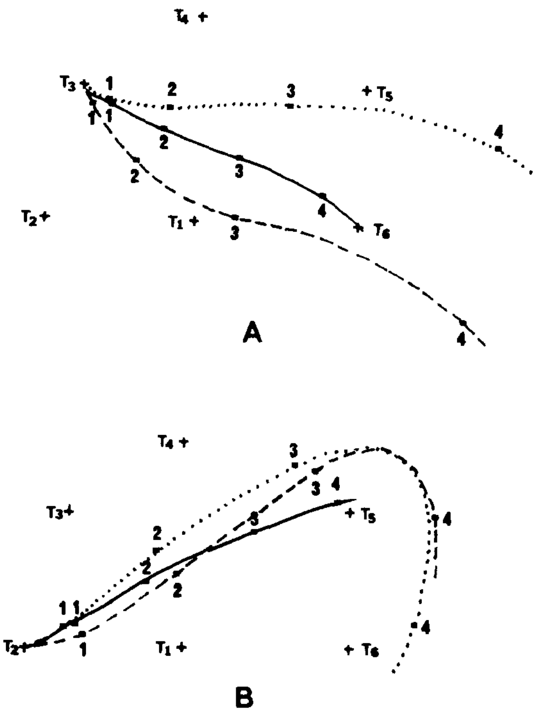

Fig. 8. Synthetic trajectories generated from a measured trajectory of the hand (solid line) from target 3 to target 6 (A) and from target 2 to target 5 (B). By removing contributions to the net joint torques due to all velocity terms (dotted line) and due to all interaction terms (dashed line), trajectories were obtained to indicate how the movement might deviate from a straight line. Corresponding points in the trajectories spaced equally in time are indicated by numbered points

for what happens to the trajectory when the interaction terms are not properly accounted for, movements were simulated with modified torques which do not contain compensation for joint interactions. These modified torques were generated from the net torques of recorded movements by removing contributions due firstly to all velocity terms and secondly to all interaction terms. The simulated trajectories which result from the modified torques are shown in Fig. 8 for the fast movements from targets 3 to 6 (A) and from targets 2 to 5 (B). As can be seen from Fig. 8, there are substantial deviations from the target and from linearity. Even more pronounced than the spatial deviation is the temporal deviation, since in the vicinity of the end point the movements are not at rest. When the torque profiles are generated without provision for interaction terms, there is no guarantee that the movement will halt at the desired end point.

4. Discussion

The results show that interaction torques are significant for different speeds of movement and for different trajectories. The velocity torques, moreover, are a non-negligible portion of the interaction torques, and, contrary to previous assumptions, their significance relative to the inertial torques does not change with movement speed. The extent of contributions from the various interaction terms, moreover, is a complicated function of the path.

Since subjects execute accurate straight line movements in these experiments, it can be concluded that the human motor system must have devised some means for precomputing or otherwise compensating for the dynamic interactions. In the remainder of this section we consider various mechanisms which have been proposed to accomodate dynamic interactions.

4.1. Open Loop Versus Closed Loop Control

The two basic alternatives for accomodating dynamic interactions are (1) an open-loop preprogram which is formed by an exact analytic computation of the joint torques, and (2) a closed-loop program consisting of a feedback controller superimposed on a preprogram with nominal torques formed from a simplified dynamics model. In the field of robotics the most common method for controlling manipulator dynamics is to generate nominal torques based on a linearization of the dynamics and to correct errors due to interaction terms and other effects by feedback. The feedback is usually structured as an independent joint controller (Golla et al., 1981), in which for purposes of design the manipulator is considered as composed of independent joints which do not interact.

While in robotics interaction terms can be compensated for by use of feedback, for biological arms feedback compensation through neuronal loops does not seem a viable option. Signals from proprioceptors are subject to many different conditions, so that their accuracy and fidelity as monitors of joint motion is a subject of current controversy. There are substantial delays in the feedback loop as well; for example, the supraspinal loop requires 70–100 ms. While the spinal loop is faster, experimental results indicate that the contribution of the spinal loop to load compensation is small (Bizzi et al., 1978). Feedback delays also limit controllable speeds of motions. If the system is changing rapidly, then by the time a feedback signal has been processed to modify the motor commands the system will have evolved to a new state for which the corrective signal is inappropriate. For fast arm movements in the range of 500–600 ms the supraspinal loop delay is too long to serve the role of a feedback controller.

A more plausible feedback mechanism has been suggested by (Hogan, 1980) and involves the regulation of the intrinsic mechanical stiffness of muscle to compensate for errors during a trajectory. The advantage of this scheme is that there is essentially no delay in generation of torques in response to deviation from a nominal trajectory. While an attractive hypothesis, the validity of this model for human arm movement has not yet been determined.

The foregoing arguments on effects of simplifying the dynamics computation and on limitations of feedback control in biological arms strongly suggest that there must exist substantially correct preprograms in order for humans to make accurate fast arm movements. Experimentally, the importance of preprogramming in the control of movement has been well established (e.g. Bizzi et al., 1976).

The two extremes of strategies for preprogramming in terms of generality and flexibility are (1) the motor tape concept and (2) real-time computation of the inverse kinematics and dynamics. The motor tape concept is that entire movements have been somehow learned and stored and that they are played back when required. Although the motor tape concept must at this point still be considered a possibility, there do seem to be severe problems with implementation, flexibility, and generality. Strategies which are intermediate to these two extremes are possible, such as final position control (Feldman, 1974), but such strategies impose some form of limitation on trajectory formation ability. No intermediate strategy has yet been put forth which has been shown capable of compensating for dynamic interactions.

The real-time computation alternative, which in principle imposes no limitations on trajectory formation ability, necessitates fast computation of the inverse dynamics. Efficient methods for computation of the inverse dynamics have received thorough study and development in robotics. Analytic solutions based either on recursive Newton-Euler formulations (Luh et al., 1980) or on recursive Lagrangian formulations (Hollerbach, 1980; Silver, 1981) offer fast, general computation of the inverse dynamics. Tabular solutions, originally proposed as plausible biological mechanisms (Albus, 1975; Raibert, 1978), trade of memory for computation by precomputing portions of the dynamic equations. These tabular solutions are not as general or accurate as the analytic solutions due firstly to the need to quantize continuous variables to fit into discrete memory, and secondly to the inability to adapt readily to mechanical changes such as when picking up an object because the masses and inertias are inextricably bound in a table. Whether the analytic or tabular solutions to dynamics computation are plausible biological mechanisms is an open question.

4.2. *The Scaling of Movement Speed*

While the dynamic requirements for different trajectories are quite varied, the exact computation of the necessary joint torques from (3) and (4) is in principle not difficult (Hollerbach, 1980). Nevertheless it is pertinent to ask whether there are any strategies of movement which cause a simplification of the dynamics computation so as to render analytic solution to (3) and (4) unnecessary or trivial. One such strategy emerged which involves a simple way of scaling movement speed.

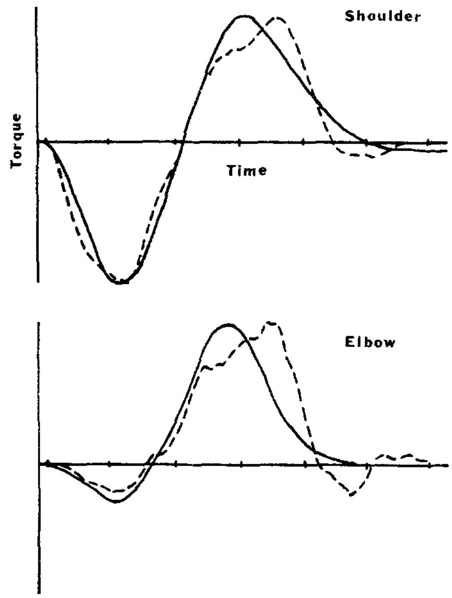

Fig. 9. Overlapped net torque profiles for the shoulder and elbow joints from fast and slow movements between targets 3 to 6. The solid line represents the 0.5 s movement, the dashed line the 1.0 s movement. The torque values of the slow movement profiles were scaled by a factor r^2 determined from the ratio of the maximum net shoulder torques for the fast and slow movements and the slow movement time axis was compressed by the same factor r

It was noted in the results section that the torque profiles remained substantially the same at different movement speeds. This result will now be explained. Newton's laws are not associated with any time scale, that is to say the rate dependent relations should hold whether the units are seconds or miliseconds. A simple scaling of movement speed should result in a simple scaling of the joint torques if gravity terms are ignored, which are not rate dependent. More precisely, suppose there is a planar trajectory described by functions of time of the joint angles $(\theta_1(t), \theta_2(t))$. A similar trajectory but at a different speed can be obtained by scaling the time by a factor r to yield $(\theta'_1(t), \theta'_2(t)) = (\theta_1(rt), \theta_2(rt))$. If $r > 1$ the movement is sped up, if $r < 1$ the movements is slowed down. The relation between the angle velocities and accelerations of the old and new trajectories for corresponding times is simply $(\dot{\theta}'_1, \dot{\theta}'_2) = r(\dot{\theta}_1, \dot{\theta}_2)$ and $(\ddot{\theta}'_1, \ddot{\theta}'_2) = r^2(\ddot{\theta}_1, \ddot{\theta}_2)$. By substituting these relations into (3) and (4) one finds that the new torques n'_1 and n'_2 are related to the old by $(n'_1, n'_2) = r^2(n_1, n_2)$.

To test this hypothesis the fast and slow movements from targets 3 to 6 illustrated in Figs. 4 and 7 were compared. The factor r^2 was determined by taking the ratio of the maximum net torque amplitudes at the shoulder for the fast and slow movements. The net torque profiles of the elbow and of the shoulder between the two movements were plotted together by compressing the time axis of slow movement by a factor r and multiplying the torques by a factor r^2. The solid lines in Fig. 9 represent the net torques of the fast movement, and the dashed lines the net torques of the slow movement scaled in the manner just described. As can be seen the profiles overlap substantially. Between these two trials therefore the subject seems to have followed a strategy of changing movement speed by scaling the time dependent torque profile. Comparisons between other movements made at different speeds substantially agree with this result.

It does not follow from the scaling observation that the human motor system is actually carrying out this scaling computation, since an alternative is that the motor system uses the same trajectory plan for different movement speeds and recomputes the dynamics each time. Nevertheless the scaling observation does increase the attractiveness of several proposed preprogramming strategies. The time domain in tabularizations of the inverse dynamics could be compressed. The motor tape concept could be made more flexible by assuming that the rate dependent portions of the torque programs are stored on a separate tape from the gravity portion of the torque programs. The rate dependent and gravity motor tapes could be combined under the scaling laws. After the rate dependent components of the torques are scaled by r^2, the gravity contribution is added in separately. While these speculations suggest ways in which the scaling law could facilitate preprogramming, whether the human motor system actually takes advantage of the scaling law must be considered at present an open question.

5. Summary

During movement of multi-jointed limbs the generation of appropriate joint torques to follow a trajectory is complicated by the presence of joint interactions due to inertial, centripetal, and Coriolis torques. These joint interactions are not present during single joint movements, which means that strategies developed for single joint movement do not necessarily generalize to multiple joint movement. The significance of the interaction terms during two joint arm movement in a horizontal plane was determined by measuring the hand trajectory with a pantograph and converting the hand positions to joint positions by solving the inverse kinematics problem. After determining the segmental parameters such as lengths and inertias, the components of joint torque were inferred by solving the inverse dynamics problem.

The results show that the interaction torques are significant relative to gravity for normal movement speeds. Additionally, the Coriolis and centripetal torques have the same significance relative to the inertial torques at all movement speeds. The velocity interaction torques in fact completely dominate the dynamics at the movement midpoint because the inertial torques go through zero as the movement switches from acceleration to deceleration and the arm is moving the fastest at this point. For movements along different straight line paths the torque profiles are quite different, which means that there is no simple way to adapt a torque program for movement along one path to generate a movement along a different path.

Since subjects normally execute straight line movements of the hand between targets, their motor control systems therefore must have computed or otherwise compensated for the dynamic interaction terms in order to maintain linearity. Simulated trajectories obtained by eliminating various interaction terms from the inferred torque profiles of measured trajectories show substantial deviation from the measured straight line paths. It was argued that computation or compensation for the dynamic interactions must already have occurred in the motor preprogram, because delays from the proprioceptors render feedback correction infeasible.

Various methods of constructing motor programs which include provisions for interaction terms have been considered, in particular analytic solutions based on recursive dynamics formulations and tabularizations of portions of the dynamic equations. A third possibility was proposed, namely that there are strategies for trajectory formation which simplify the dynamics computation. By scaling the time dependent portions of the joint torque profiles by a factor r^2 in order to speed up a movement by a factor r and then factoring in gravity separately, it is possible to change movement speed without deviating from the desired path. If one knows one particular way of making a movement between two points, therefore, it is possible to make movements between the same two points at different speeds without a substantial dynamics recomputation. Comparisons of human arm movements between the same targets but at different speeds show that subjects evidently adapt a strategy compatible with this hypothesis to alter movement speed.

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