

## Fitts' Law in Two Dimensions with Hand and Head Movements

Richard J. Jagacinski  
Ohio State University

Donald L. Monk  
Air Force Aerospace Medical Research Laboratory  
Wright-Patterson Air Force Base

---

**ABSTRACT.** Subjects performed two-dimensional discrete movements either with a helmet-mounted sight or with a joystick. Fitts' Law was found to be a good predictor of the speed-accuracy tradeoff for both systems. The joystick produced faster movement times than the helmet-mounted sight. For both systems, horizontal and vertical movements were slightly faster than diagonal movements. Two dimensional generalizations of Fitts' Law were discussed in terms of multidimensional scaling. The obtained pattern of movement times was found to be intermediate to the predictions of Euclidean and City-block models of the movement space. Muscle coordination strategies were considered, and a strictly serial coordination model was rejected. A strictly parallel model was also rejected for the helmet-mounted sight, but not for the joystick.

---

THE TIME TO MOVE from a home position to a stationary target of prespecified width has generally been found to be a logarithmic function of the required movement accuracy. Namely,

$$MT = a + \text{blog}_2 \left( \frac{A}{W/2} \right) \quad (1)$$

where  $MT$  is movement time,  $A$  is the distance from the home position to the target center, and  $W$  is the target width. The ratio  $A/(W/2)$  is the

---

*This work was conducted with personnel of Systems Research Laboratories, Inc. under Contract F33615-C-79-0503, with the Air Force Aerospace Medical Research Laboratory, Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under Project 6893. Portions of this work were reported at the 18th Annual Conference on Manual Control, Dayton, Ohio, June 1982.*

*The authors wish especially to thank Grant McMillan, Jim Porterfield, Bob McMurry, Jeff Heckerson, Bob Smith, Curtis Wray, and Matt Middendorf for their assistance in executing and analyzing the present experiment.*

*Requests for reprints should be sent to Donald L. Monk, Air Force Aerospace Medical Research Laboratory (AFAMRL/HEC), Wright-Patterson Air Force Base, Ohio 45433. Further reproduction is authorized to satisfy needs of the U.S. Government.*

required accuracy of the movement, and the speed-accuracy relationship represented by Equation 1 is referred to as Fitts' Law (Fitts & Peterson, 1964).

Although by no means the only empirical speed-accuracy tradeoff for discrete movements (eg., see Schmidt, Zelaznik, Hawkins, Frank & Quinn, 1979; Howarth, Beggs, & Bowden, 1971), Fitts' Law has received a good deal of attention partially because of its applicability to industrial assembly tasks (Langolf, Chaffin, & Foulke, 1976), vehicular control (Drury, 1975; Hartzell, Dunbar, Beveridge, & Cortilla, 1982), and military and industrial target acquisition tasks (Card, English, & Burr, 1978; Jagacinski, Repperger, Moran, Ward, & Glass, 1980). Another reason for this attention is the generality of this speed-accuracy relationship across a wide range of muscle groups and movement amplitudes (Drury, 1975; Langolf et al., 1976). The present experiment attempted to test further the generality of this relationship in two ways. First, does Fitts' Law generalize to head movements? The use of head movements has received attention both for aircraft pilots faced with many simultaneous control tasks (Barnes & Sommerville, 1978) and for handicapped individuals who have lost control of their hands and arms (Soede, Stassen, van Lunteren, & Luitse, 1973). Levison, Zacharias, Porterfield, Monk, and Arbak (1981) recently found that for a continuous pursuit tracking task, manual and head tracking had very similar structure, once the dynamics of the head were taken into account. The present experiment compared discrete manual and head movements in a target acquisition task. Subjects used either a joystick or a helmet-mounted sight to control a cursor displayed on a CRT screen. Their task was to capture a circular target by positioning the cursor over the target as rapidly as possible. The question of interest was whether Fitts' Law would generalize to head movements.

A second issue in this experiment was whether Fitts' Law would generalize to two-dimensional movements. In the present experiment, angular uncertainty was varied over  $360^\circ$ . Targets could randomly appear along any of eight radii covering a circular two-dimensional region (Figure 1). Of interest was whether a two-dimensional generalization of Fitts' Law could be found over this range. In previous experimentation, subjects' uncertainty as to movement direction has generally been restricted to two radii extending from the home position and differing by  $180^\circ$ . Target location was thus restricted along a unidimensional axis on any given trial. One exception was a study of CRT text selection by Card et al. (1978) in which the target could appear over a  $90^\circ$  quadrant. Fitts' Law provided a good approximation to movement times with a mouse (a small device rolled on a flat surface to produce corresponding movement of a CRT cursor) and with a nonlinear isometric rate-controlled joystick. There were, however, some systematic deviations from Fitts' Law for the latter controller. Movement angle did not have a statistically significant effect with the mouse. With the joystick, diagonal movements took longer by approximately 3% of the overall mean movement time.

Two examples of possible two-dimensional generalizations of Fitts' Law based on multidimensional metric scaling (Coombs, Dawes, & Tver-

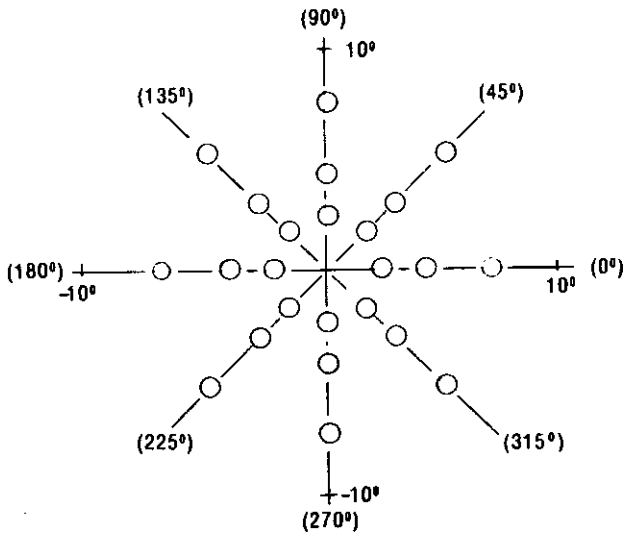


Fig. 1—Twenty-four target locations along eight radii. Distance from the center starting position is in degrees of visual angle.

sky, 1970) are shown in Figure 2. In this Figure the horizontal and vertical coordinates of the two-dimensional target space are respectively referred to as  $x$  and  $y$ . The distance from the home position to the target center can then be described in terms of horizontal and vertical components  $A_x$  and  $A_y$ . If these two components are combined by a Euclidean combination rule, the effective movement amplitude is  $(A_x^2 + A_y^2)^{1/2}$ . According to this rule, the effective movement amplitude is the shortest physical distance between the home position and target center. Movement direction is therefore unimportant in determining movement time; diagonal, horizontal, and vertical movements of the same Euclidean amplitude  $A$  will all be performed in the same amount of time. The City-block combination rule is also shown in Figure 2. According to this rule, the effective movement amplitude is  $A_x + A_y$ , analogous to the distance one would have to travel in a city in which the streets ran along the  $x$  and  $y$  dimensions. Consistent with the analogy of moving through an array of city blocks, movement is more efficient when it is along only one of the two principal axes,  $x$  or  $y$ . The two combination rules in Figure 2 are by no means exhaustive, but they do illustrate possible two-dimensional generalizations of Fitts' Law that reduce to the usual one-dimensional formulation if either  $A_x$  or  $A_y$  is set to zero. Similar combination rules can also be hypothesized for the horizontal and vertical dimensions of the target,  $W_x$  and  $W_y$ . However, only circular targets were used in the present experiment, so that target width is represented by a single parameter,  $W$ , in Figure 2.

The comparison of the Euclidean and City-block metrics highlights the question of whether there are particular axes along which movement is

most efficient. Previous research on blind positioning movements (Fitts, 1947, cited in McCormick & Sanders, 1982), visually guided positioning movements (Siddall, Holding, & Draper, 1957), and movements at constant velocities (Corrigan & Brogden, 1949; Keele, 1968) have shown certain spatial directions to have greater efficiency. Therefore, it would not be surprising to find directions of greater and lesser efficiency in the present experiment. If one assumes that a discrete set of muscle groups is used to perform two-dimensional movements, then such axes might result from the geometric orientation of these muscle groups. For example, Figure 3 shows a horizontal agonist-antagonist muscle group (1 and 1') and a similar vertical muscle group (2 and 2'). These muscle groups are schematically represented as pairs of opposing springs (Asatryan & Fel'dman, 1965). For a horizontal or vertical movement, only one pair of springs needs to be activated. However, for diagonal movements, both pairs must be activated.

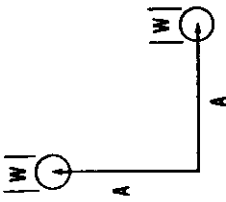
Two extremes in the many ways the two muscle groups might be activated are serial and parallel activation (Figure 3). Serial activation represents a minimal kind of coordination in which first one muscle group is activated and then the other. A diagonal target would be reached by a horizontal movement followed by a vertical movement, or vice versa. The effective amplitude for those horizontal and vertical components is  $0.707A$ , and the effective target width is  $0.766W$  as can be argued as follows. For unidimensional horizontal or vertical movement,  $W$  can be considered to represent a 90% confidence interval over an approximately normal distribution of movement ending points. In other words,  $W$  is  $2 \times 1.645\sigma = 3.290\sigma$ , where  $\sigma$  is the standard deviation of the normal distribution. If the normal distribution of ending points for the horizontal and vertical components of a diagonal movement are uncorrelated and have a common standard deviation,  $\sigma$ , then the combined distribution is bivariate normal and circular in shape. A 90% confidence circle corresponding to  $W$  will have a radius of  $2.146\sigma$ , since the square of the radius is a chi-square variable with two-degrees of freedom. Therefore,  $W$  corresponds to  $2 \times 2.146\sigma = 4.292\sigma$ , or  $\sigma = W/4.292$ . The effective target width for the horizontal and vertical components is therefore  $3.290 \times W/4.292 = 0.766W$ . In other words, diagonal movement to a target of diameter  $W$  corresponds to an effective target width of  $0.766W$  for the horizontal and vertical components, if the subject's movement trajectories are to terminate inside the target circle 90% of the time. If the movement times for the horizontal and vertical components are described by Equation 1, then the total movement time for serial activation would be

$$MT = 2 \left\{ a + b \log_2 \left[ \frac{2(.707A)}{.766W} \right] \right\} \quad (2)$$

$$MT = 2 \left\{ a + b \log_2 \left( \frac{2A}{W} \right) \right\} - .231b \quad (3)$$

$$MT = 2MT_{h-v} - .231b \quad (4)$$

HORIZONTAL OR  
VERTICAL MOVEMENT



EUCLIDEAN COMPOSITION RULE

$$MT = a + b \text{ Log}_2 \frac{2(A_x^2 + A_y^2)^{1/2}}{W}$$

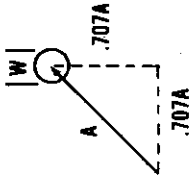
$$MT = a + b \text{ Log}_2 \frac{2A}{W}$$

CITY - BLOCK COMPOSITION RULE

$$MT = a + b \text{ Log}_2 \frac{2(A_x + A_y)}{W}$$

$$MT = a + b \text{ Log}_2 \frac{2A}{W}$$

DIAGONAL MOVEMENT



$$MT = a + b \text{ Log}_2 \frac{2A}{W}$$

$$MT = a + b \text{ Log}_2 \frac{2(1.414A)}{W}$$

$$MT = a + b \text{ Log}_2 \frac{2A}{W} + .5b$$

Fig. 2—Euclidean and City-block combination rules for movement time,  $MT$ , in terms of horizontal ( $x$ ) and vertical ( $y$ ) coordinates.

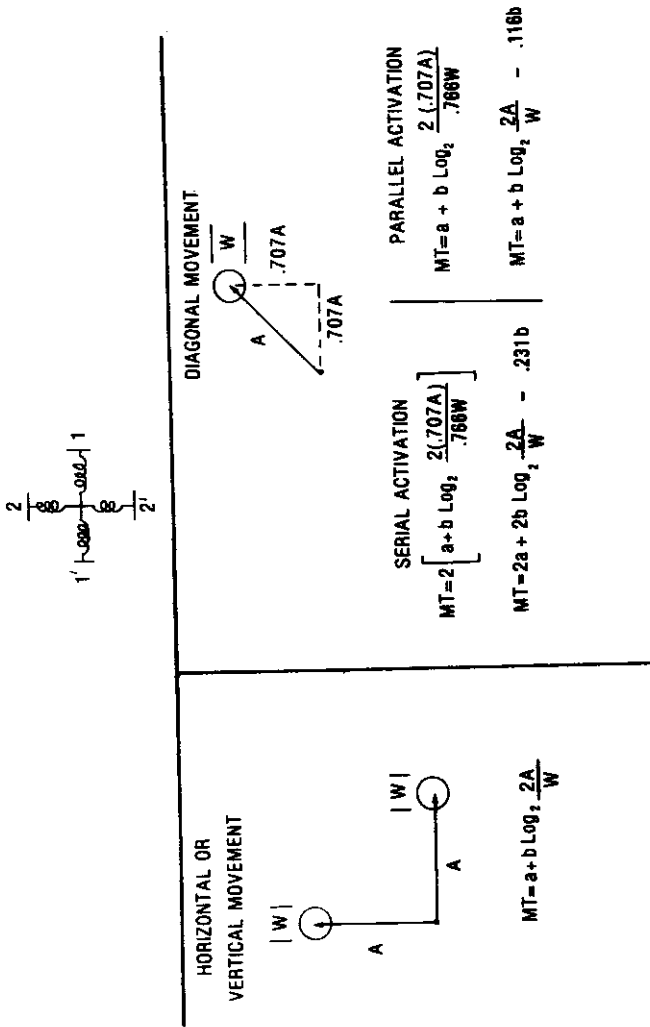


Fig. 3.—Movement times for serial and parallel coordination schemes.

$MT_{h-v}$  is the movement time for a horizontal or vertical movement with amplitude  $A$  and target width  $W$ . If  $MT_{h-v}$  is larger than 0.231 times the slope,  $b$ , for all combinations of  $A$  and  $W$  then the serial activation model predicts longer movement times for diagonal movements. This analysis assumes that horizontal and vertical movements can be executed independently with negligible movement on the other axis.

In the case of parallel activation, the two muscle groups are activated simultaneously. The movement time for the diagonal movement is then

$$MT = a + b \log_2 \left[ \frac{2(.707A)}{.766W} \right] \quad (5)$$

$$MT = a + b \log_2 \left( \frac{2A}{W} \right) - .116b \quad (6)$$

which is  $0.116b$  less than the time for a horizontal or vertical movement with the same values of  $A$  and  $W$ . At first this prediction seems counter-intuitive in that activating more muscle groups would be expected to have some dual-task decrement. However, the assumption of parallel activation is essentially one of perfect coordination. The present experiment tested these serial and parallel coordination models and more generally examined the spatial symmetry of a speed-accuracy tradeoff.

## Method

### Subjects

Subjects were seven male and one female employees of Systems Research Laboratories, who had participated previously in tracking experiments. They ranged in age from 20 to 28 years. All subjects were right-handed and were pretested to determine that they expected to push forward on the joystick to make the cursor move upward. Directional incompatibility with the joystick control was thus avoided.

### Apparatus

For the manual capture task, subjects used a Measurement Systems 525 X-Y potentiometer joystick. The joystick shaft was 6 cm long, 0.4 cm in diameter, and not spring centered. The range of travel was  $45^\circ$  away from the center position in all directions, and the gain was set at the low value of  $0.34^\circ$  visual angle per  $1^\circ$  of joystick movement. An experiment by Gibbs (1962) suggests that low gains produce faster capture times for manual position control systems with no lag. The orientation of the joystick was such that the joystick was vertical when it was centered in both dimensions. There was negligibly small hardware noise in both axes (standard deviation =  $0.01^\circ$  visual angle).

For the head movement capture task, subjects wore a 1.5 kg helmet-mounted sight that used two rotating infrared-beams for the measurement of head position. The X-Y position of the head was updated at a 30 Hz rate by this system. Its hardware noise had an approximately Gaussian amplitude distribution and 6 Hz bandwidth. However, the standard

deviation of the amplitude distribution was  $0.07^\circ$  visual angle in the horizontal axis and only  $0.02^\circ$  visual angle in the vertical axis. In order to avoid possible asymmetries in capture times due to this difference in noise amplitude, additional noise of 6 Hz bandwidth was added to the vertical axis to equate it with the horizontal axis. The gain of the helmet-mounted sight was  $1^\circ$  visual angle per  $1^\circ$  of head rotation so that the required movement corresponded to turning one's head directly toward the target. Scaling and zeroing of both the helmet-mounted sight and the joystick were accomplished through an Electronic Associates TR-20 analog computer.

The targets were displayed as circles on a Hewlett Packard 1310A X-Y display with P4 phosphor. The circles were generated by an IMLAC PDS-4 graphics terminal under the control of a PDP 11/34 computer. This latter computer also sampled and stored the outputs of the joystick and helmet-mounted sight at a 90 Hz rate after the signals had passed through 15 Hz Butterworth filters. These signals were always displayed to the subject as a single-dot cursor on the X-Y display. The aiming reticle feature of the helmet-mounted sight was not used.

### *Procedure*

Seventy-two stationary circular targets were created from a factorial crossing of eight radii, three movement amplitudes (*A*), and three target diameters (*W*). The eight radii corresponded to  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$ ,  $180^\circ$ ,  $225^\circ$ ,  $270^\circ$ , and  $315^\circ$ .  $0^\circ$  corresponded to the three o'clock position, and the angles increased counterclockwise (Figure 1). The three movement amplitudes were  $2.45^\circ$ ,  $4.28^\circ$ , and  $7.50^\circ$  visual angle as measured from the center of the X-Y display to the center of the target circles. The three target diameters were  $0.30^\circ$ ,  $0.52^\circ$ , and  $0.92^\circ$  visual angle for the joystick, and  $0.40^\circ$ ,  $0.70^\circ$ , and  $1.22^\circ$  visual angle for the helmet-mounted sight. The larger target widths were used for the helmet-mounted sight because of its greater hardware noise.

At the start of a trial the smallest diameter target appeared at the center of the X-Y display. The subject controlled the position of a cursor, which appeared as a small dot. In order to begin a trial, the subject first had to hold the cursor within the centering circle for 330 ms. When this criterion was achieved, the centering circle was replaced by a medium diameter circle for 250 ms as a warning circle. Then, the smallest diameter circle appeared again for 250 ms. Provided the cursor stayed within the centering circle for this latter 250 ms interval, the trial began. If the cursor drifted out of the centering circle, the cursor had to return for a continuous 250 ms interval before the trial began. The medium diameter warning circle did not reappear on these trials delayed by drift.

A trial began with the disappearance of the centering circle and the simultaneous appearance of one of the 72 targets. The subject's task was to move the cursor within the target circle and hold it there for a period of 344 ms, of which no more than 67 ms could be spent momentarily outside the target circle. When this criterion was met, the target was



captured, and it disappeared from the X-Y display. The 67 ms aspect of the capture criterion avoided penalizing the subject for slight amounts of jitter. Four seconds after a target was captured, a new trial began with the appearance of the centering circle.

Three temporal measures of performance were recorded for each trial. Reaction time was defined as the period beginning with the disappearance of the centering circle and ending when the distance between the cursor and the center of the X-Y display exceeded the radius of the medium size warning target circle. The medium size circle was chosen as the criterion rather than the smallest circle to avoid mistaking a drift in cursor position for the beginning of a capture movement. Capture time was defined as the period beginning with the appearance of the target and ending with the start of the 344 ms capture period. Movement time was defined as the difference between capture time and reaction time.

### *Design*

Each experimental session consisted of nine practice trials followed by 108 data trials. After a two-minute break, there were another nine practice trials and another 108 data trials. The 216 data trials consisted of a random ordering of three replications of each of the 72 targets. At the end of each session, subjects were told their mean capture time for that session to encourage motivated performance. Also, at the beginning of each session, subjects were shown a graph of their daily capture times up to that point, and also the capture times of the other subjects.

Each of the eight experimental subjects used both the joystick and the helmet-mounted sight. Four subjects were randomly chosen to start with the joystick, and the other four subjects started with the helmet-mounted sight. Subjects practiced with their assigned control system until they reached a performance asymptote. Asymptote was defined as four consecutive days in which the mean capture time for each of the days did not differ from the overall mean across days by more than 3.5%. After reaching asymptote, the subject was transferred to the other control system and again practiced until reaching asymptotic performance.

## **Results**

### *Applicability of Fitts' Law*

Subjects took from six to eighteen days to reach asymptotic performance with the manual joystick and from seven to twenty-nine days with the helmet-mounted sight. For each subject's last four days of performance with each control system, a daily median reaction time, movement time, and capture time were calculated for each of the 72 different targets. For each of the nine combinations of movement amplitude,  $A$ , and target diameter,  $W$ , means of these medians were calculated across eight different radii, the last four days of performance, and the eight subjects. Mean movement times are shown in Figures 4 and 5. Each data point is the mean of  $8 \times 4 \times 8 = 256$  medians, and each median is based

on three trials. Fitts' Index of Difficulty,  $\log_2(2A/W)$ , was found to be a good linear predictor of the joystick movement times ( $r = .98$ ), but a somewhat poorer predictor of the helmet-mounted sight movement times ( $r = .93$ ). The major difficulty with these latter data was that the movement times for the smallest target diameter were surprisingly long (Figure 4).

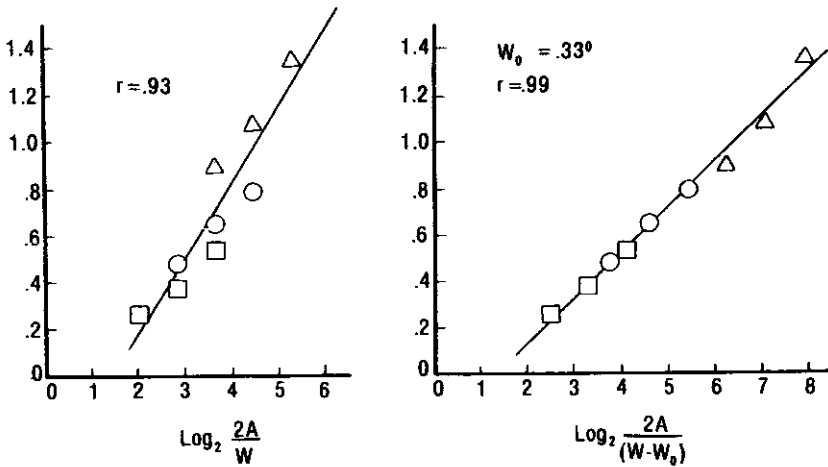
One modification of the Index of Difficulty that might improve the linear fit to these data is to represent the effects of the hardware noise in each control system. Namely, the cursor position randomly varies over a small, approximately circular distribution due to random perturbations in the control system hardware. The subject's task is thus somewhat analogous to placing a peg in a hole, except that the "peg" in this case is the result of a two-dimensional noise distribution rather than being a physically constant peg. Researchers investigating the applicability of Fitts' Law to peg insertion tasks have previously taken the effective target diameter to be the difference between the peg diameter,  $W_p$ , and the nominal target diameter,  $W$  (Fitts, 1954; Langolf et al., 1976). The modified Index of Difficulty is then  $\log_2[2A/(W - W_p)]$ . For the present task the "peg" diameter,  $W_p$ , was defined as the smallest diameter target that theoretically could be captured on 90% of the trials if the subject perfectly positioned the cursor at the target center. The values for  $W_p$  were  $0.33^\circ$  for the helmet-mounted sight and  $0.07^\circ$  for the joystick. The resulting movement time correlation coefficients obtained from this modified Index of Difficulty were .99 for both the joystick and helmet-mounted sight (Figures 4 and 5).

A conservative test of the statistical significance of these increased correlations can be obtained by treating  $W_p$  as though it were a fitted parameter rather than a parameter derived from the characteristics of the system hardware. An  $F$  test for extra degrees of freedom can then be performed. The numerator is the reduction of the residual variance obtained with the parameter  $W_p$ , divided by the additional degrees of freedom (i.e., one). The denominator is the residual variance with the more elaborate Index of Difficulty, divided by its degrees of freedom (i.e., six). The improvement in the correlation coefficient was found to be statistically significant ( $p < .05$ ) for both the joystick and the helmet-mounted sight. The modified Index of Difficulty was therefore used in all subsequent analyses.

The regression equations for the means of the median reaction times (RT), movement times (MT), and the total capture times (CT) are shown in Table 1. The movement time equations are for the data in the right panels of Figures 4 and 5. The reaction time regression equations had zero slopes. The capture time regression equations had approximately the same slopes as the movement time equations.

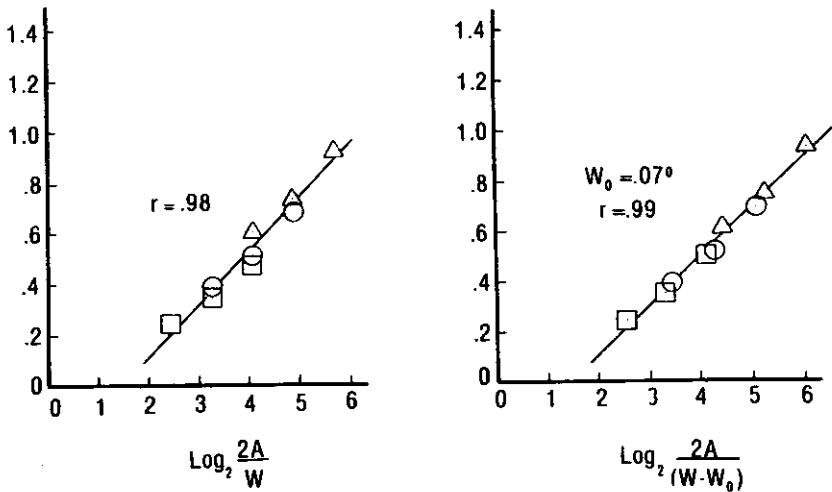
#### *Alternative Metrics for Movement Space*

The initial analysis revealed that a modified version of Fitts' Index of Difficulty was a good predictor of movement times and capture times aver-



HELMET - MOUNTED SIGHT

Fig. 4—Mean movement times with the helmet-mounted sight for each of nine combinations of movement distance, A, and target diameter, W, averaged across the eight radii. Triangles, circles, and squares respectively refer to 0.40°, 0.70°, and 1.22° target diameters.



JOYSTICK

Fig. 5—Mean movement times with the joystick for each nine combinations of movement distance, A, and target diameter, W, averaged across the eight radii. Triangles, circles, and squares respectively refer to 0.30°, 0.52°, and 0.92° target diameters.

**Table 1**  
**Regression Equations for All Eight Radii**

Helmet-mounted sight		
$RT = .355 + .000 \log_2 [2A/(W - .33)]$		$r = .09$
$MT = -.268 + .199 \log_2 [2A/(W - .33)]$		$r = .99$
$CT = .085 + .200 \log_2 [2A/(W - .33)]$		$r = .99$
Joystick		
$RT = .250 + .000 \log_2 [2A/(W - .07)]$		$r = .00$
$MT = -.303 + .199 \log_2 [2A/(W - .07)]$		$r = .99$
$CT = -.053 + .199 \log_2 [2A/(W - .07)]$		$r = .99$

aged across eight radii. A second issue of interest is whether or not some radii permit more efficient movements than others. An Euclidean movement space would have all radii of equal difficulty; a City-block movement space would have one orthogonal set of dimensions be most efficient.

In order to test these different possible structures of the movement space, the three temporal measures, reaction time, movement time, and capture time, were analyzed separately for two sets of radii. The two horizontal and two vertical radii formed one set, and the four diagonal radii formed the other set. For each subject, median times for each of the nine combinations of movement amplitude,  $A$ , and target width,  $W$ , were averaged across the last four days of performance and across the four radii in each set. Regression lines were fit to each subject's data for each set of radii. Correlation coefficients ranged from  $-.48$  to  $.55$  for reaction times,  $.93$  to  $.99$  for movement times, and  $.93$  to  $.99$  for capture times. A  $t$  test for paired comparisons was performed on the slopes of the regression lines and the mean times for each of the three temporal measures. Means were tested rather than intercepts because the mean is statistically independent from the slope of the regression line and has a smaller standard error than the intercept (Draper & Smith, 1966).

The results of these analyses are summarized in Table 2, which shows the average (mean) values of slopes and means across the eight subjects. There were no significant differences in slope between the horizontal-vertical radii and the diagonal radii for any of the three temporal measures. For the helmet-mounted sight, mean movement times were  $0.062$  s longer, and mean capture times were  $0.066$  s longer for the diagonal radii ( $p < .05$ ). For the joystick, mean reaction times were  $0.003$  s shorter, and movement times were  $0.038$  s longer for the diagonal radii ( $p < .05$ ). The mean capture time difference for the joystick ( $0.035$  s) just missed statistical significance ( $t = 2.28$ ), apparently due to the difference in sign between the reaction time and movement time effects. Figure 6 shows mean movement time as a function of the eight different radii. Overall, these results indicate that movements along the horizontal-vertical radii tended to be more efficient than movements along the

**Table 2**  
**Performance Along The Horizontal-Vertical and Diagonal Axes**

	Slopes (s/bit)		
	RT	MT	CT
<b>Helmet-mounted sight</b>			
Horizontal-vertical	.000	.200	.200
Diagonal	.000	.198	.199
<b>Joystick</b>			
Horizontal-vertical	.000	.203	.202
Diagonal	.000	.195	.195
	Means (s)		
	RT	MT	CT
<b>Helmet-mounted sight</b>			
Horizontal-vertical	.355	.682	1.035
Diagonal	.360	.744*	1.101*
<b>Joystick</b>			
Horizontal-vertical	.252	.526	.778
Diagonal	.249*	.564*	.813

\* Significantly different from horizontal-vertical,  $p < .05$

diagonal axes for both the helmet-mounted sight and the joystick. This tendency was somewhat more pronounced for the helmet-mounted sight. These results reject the Euclidean model of a uniformly efficient movement space.

The City-block model of the movement space states that there is a set of most efficient movement axes. If these axes are taken to be the horizontal and vertical axes in the present experiment, then the movement times for the diagonal axes should (a) have the same slope as the horizontal-vertical axes, and (b) have a mean that is longer by one-half the slope (Figure 2). The results in Table 2 indicated that there were no significant differences in the movement time slopes between the horizontal-vertical radii and the diagonal radii. Thus condition (a) was satisfied within the limits of statistical power for this experiment. Condition (b) was tested by multiplying the slope of the movement time regression line for all eight radii by one-half and subtracting it from the difference between the mean diagonal and mean horizontal-vertical movement times for each subject. If the City-block model is correct, the result of this calculation should be approximately zero. However, a  $t$  test revealed that across subjects the resulting calculation tended to be different from

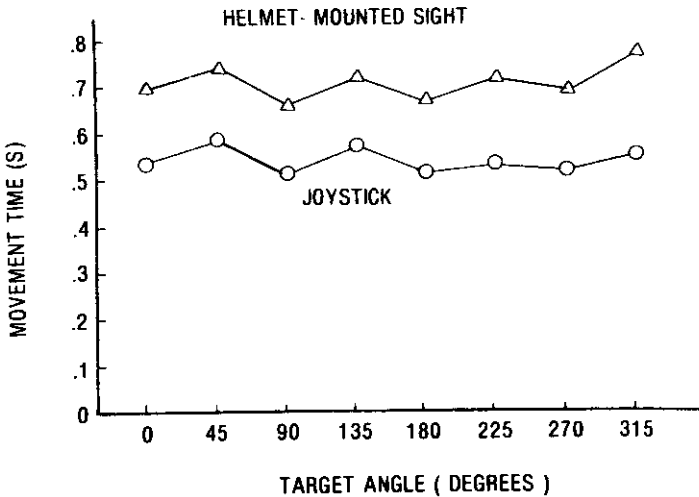


Fig. 6—Mean movement time as a function of the eight radii.

zero for both the helmet-mounted sight and the joystick ( $p < .05$ ). The City-block model predicts about a 0.100 s mean difference between the diagonal and horizontal-vertical axes for both the helmet-mounted sight and joystick. The obtained differences (0.062 and 0.038 s) are smaller than this prediction. Therefore, the City-block model must also be rejected.

The serial activation model can be rejected for the present data. Inspection of movement trajectories revealed that diagonal targets are not approached via a single horizontal movement followed by a single vertical movement (or vice versa), so the serial model is rejected. If the muscle groups are taken as shown in Figure 3, then the parallel activation model is rejected because the diagonal movements were found to be longer rather than shorter than horizontal and vertical movements. However, if the muscle groups are aligned diagonally rather than vertically as in Figure 3, then the prediction of the parallel activation model is that the diagonal movement times will be longer by 0.116 times the movement time slope. A  $t$  test similar to that used for testing the City-block model indicated that this version of the parallel activation model could be rejected for the helmet-mounted sight ( $p < .05$ ), but not for the joystick ( $t = .099$ ). This model predicts about a 0.023 s mean difference between the diagonal and horizontal-vertical axes for both the helmet-mounted sight and joystick.

## Discussion

### *Applicability of Fitts' Law*

*Generalities.* The present experiment has helped to generalize the applicability of Fitts' Law in three ways. First, Fitts' Law has been found to

be a useful predictor of target acquisition times in a two-dimensional array in which angular uncertainty varied over  $360^\circ$ . The finding of slightly longer diagonal movement times is consistent with the results of Card et al. (1978) for the rate-controlled joystick, but not for the mouse. In another previous study Jorgeson (1966) working with Pew varied the angle relative to the body with which subjects performed blocks of one-dimensional horizontal arm movements. They found that mean movement times increased as the angle changed counterclockwise from  $0^\circ$  to  $180^\circ$  (Figure 1). This pattern is quite different from the one found in the present experiment, and the reasons for this difference are presently unknown.

Secondly, the present results generalize previous analyses of peg insertion tasks to systems in which the "peg" of diameter  $W_0$  is formed by a stochastic noise distribution. Hardware noise can be considered as decreasing the effective target width to  $(W - W_0)$ , and use of the effective target width in the Index of Difficulty significantly improved the linear prediction of target capture times.

Thirdly, the present results indicate that target acquisitions performed with head movements are well described by Fitts' Law. Previously Fitts' Law has been found to hold for arm, wrist, and finger movements (Langolf et al., 1976) and foot positioning movements primarily involving the lower leg (Drury, 1975). The generality with regard to the particular limb indicates that the basis for Fitts' Law cannot be muscle group specific.

Although Fitts' Law was found to hold for both head and hand movements, there was a major difference in performance levels. The reaction times, movement times, and capture times for the helmet-mounted sight were all longer than the corresponding measures of joystick performance. These results parallel the findings of Levison et al. (1981) for continuous pursuit tracking. Namely, they found striking similarities in the structural aspects of manual and head tracking; however, the overall performance level for manual tracking was superior. The present results should not, however, be regarded as indicative of the best performance possible with head movements. First of all, the helmet-mounted sight in the present experiment was quite heavy, and lighter weight helmets might well yield faster capture times. For example, Fitts (1954) found that increasing the weight of a stylus increased the time to perform arm movements. Secondly, the subjects in the present experiment all had previous experience with laboratory manual tracking tasks, but none had comparable experience with head tracking. Two subjects who took 26 and 29 days to reach asymptotic performance with the helmet-mounted sight had mean capture times of 0.842 s and 0.865 s over their last four days of performance, which were very close to their respective joystick capture times of 0.725 s and 0.906 s. Although one cannot draw any firm conclusions from these observations, further investigation of more extended practice appears necessary before estimating the absolute performance levels possible with a helmet-mounted sight.

*Limitations.* Despite these generalities with regard to Fitts' Law, several limitations of these findings should be considered. First, the targets used in this experiment were stationary rather than moving targets. Jagacinski,

Repperger, Ward, and Moran (1980) have shown Fitts' Law does not hold for the capture of one-dimensional moving targets with a position control system, and similar limitations may apply in two dimensions.

Secondly, the present experiment specified the required target width and measured movement time as a dependent variable. Experiments in which desired measurement time has been specified by the experimenter and effective target width,  $W_e$ , measured as a dependent variable (e.g., Schmidt et al., 1979) have found movement time to be a linear function of  $A/W_e$ , rather than a logarithmic function. Meyer, Smith, and Wright (1982) have attempted to incorporate both of these speed-accuracy relations in a theory of discrete movements. Similar integrations with the results of Howarth et al. (1971) would contribute toward a more general theory of speed-accuracy tradeoffs for discrete movements.

Thirdly, Fitts' Law may not hold for Indices of Difficulty very much larger or very much smaller than those used in the present experiment. For example, Crossman and Goodeve (1963) and Klapp (1975) showed that for targets having a very low Index of Difficulty, movement times eventually reached an asymptotic lower bound. For targets of very large Index of Difficulty one would expect tremor to become an increasingly important factor, and movement times might begin to rise very rapidly. However, this latter point remains to be tested. Subject to these limitations, the present experiment has found Fitts' Law to be a good predictor of the time to capture stationary targets of specified width in a range of required movement accuracies from roughly two to eight bits.

### *Metric Descriptions of Movement Space*

Both the helmet-mounted sight and the joystick control systems had slightly faster movement times along the horizontal and vertical axes than along the diagonal axes. This nonuniformity rejects the Euclidean model for two-dimensional movement in Figure 2. The sizes of the effect, 0.038 s for the joystick and 0.062 s for the helmet-mounted sight, were not as large as predicted by the City-block model in Figure 2. While these effects are relatively small, they may be of considerable theoretical interest.

The source of the nonuniformity of movement efficiency could be asymmetries in the laboratory hardware or asymmetries in the perceptual and/or motor systems of the subjects. Hardware asymmetries could be attributed to two sources. The joystick construction involved lateral and fore-aft axes of rotation, which respectively correspond to the horizontal and vertical axes of the visual display of the cursor and target. Diagonal targets required movements about both joystick axes. Although mechanical resistance felt negligible, it is possible that it was slightly greater for diagonal movements. However, if this were the case and the effect was large enough to influence performance, one would also expect longer reaction times along the diagonal axes. In contrast to this expectation, joystick reaction times were very slightly shorter along the diagonal axes; therefore, this explanation seems unlikely.



For the helmet-mounted sight, the asymmetric shape of the helmet plus head might cause the moment of inertia to be greater along the diagonal axes. Once again, if this effect were large enough to affect performance, one would expect longer diagonal reaction times. Diagonal reaction times were in fact 0.005 s longer, although this difference was not statistically significant. Formal measurements of moments of inertia are necessary to further explore this possible explanation.

A second possible explanation is that perceptual factors were responsible for longer movement times along the diagonal axes. For example, if visual acuity was superior in the horizontal and vertical axes, this factor might account for the present results. Assessments of visual acuity over the two-dimensional space have apparently not yet been performed, so this point is at present a matter of speculation.

A third possible explanation is that motor programming is more efficient along the horizontal and vertical axes. For the joystick system, reaction times along the diagonal axes were 0.003 s faster. This result might indicate that the initial submovement was programmed in less detail (Klapp, 1975), with a shorter duration (Quinn, Schmidt, Zelaznik, Hawkins, & McFarquhar, 1980; Klapp, 1981), and/or with a higher velocity (Falkenburg & Newell, 1980) for the diagonal movements. However, these explanations could not apply to the helmet-mounted sight system, where no statistically significant reaction time differences were found, and the diagonal reaction time was 0.005 s longer than the horizontal-vertical reaction time.

Another type of motoric explanation concerns the details of coordinating different sets of muscles. Assuming two orthogonal pairs of muscle groups, a serial activation model was rejected for both the helmet-mounted sight and the joystick. A parallel activation model was rejected for the helmet-mounted sight, but not for the joystick. Combinations of parallel and serial muscle activation also cannot be rejected. However, more direct measurements of muscle activation and the geometric orientation of muscle groups are necessary to explore why movement along the horizontal and vertical axes is more efficient. It should be noted that this pattern of results assumed a 90% confidence interval for hardware noise in defining  $W_0$ , and interpreted the effective target width,  $W - W_0$ , as a 90% confidence interval on the distribution of movement "endpoints." In that all of the trials eventually resulted in target capture, this latter interpretation assumes that the movement control processes described by Fitts' Law bring the cursor within  $(W - W_0)/2$  of the target center on 90% of the trials. For those trials in which either the movement control processes or unusually large hardware noise causes the cursor to exceed the target boundary late in the capture process, additional corrections will be necessary, and will lengthen the measured movement time and capture time if the 67ms criterion is exceeded. However, these infrequent trials were assumed to be effectively screened from the overall time measures by the use of medians across trials in the present data analysis. More detailed modeling of the interac-

tion of hardware noise, motoric unsteadiness, and possible corrective movements during the capture criterion interval remains to be pursued.

In summary, the present experiment examined both the applicability of Fitts' Law and the uniformity of this relation over a two-dimensional target space. Once multidimensional movement is considered rather than unidimensional movement, then many of the structural issues associated with multidimensional perceptual research can be raised in a new domain. For example, the Euclidean and City-block combination rules are examples of Minkowski metrics (Coombs et al., 1970), which have received a good deal of attention in research on multidimensional aspects of perception. Researchers have been interested in multidimensional spatial representations of the similarity among stimuli. Of particular interest has been whether such a space was best described in terms of a particular set of most salient dimensions, or whether the space was uniform (Shepard, 1964; Pachella, Somers, & Hardzinski, 1981). The present experiment raised a similar question with regard to the speed-accuracy characteristics of discrete movements and has raised interesting structural questions regarding the underlying motor and perceptual processes. This approach is not limited to the Fitts' Law paradigm. Similar issues can be raised within the contexts of other discrete and continuous movement paradigms as well. For example, Navon, Gopher, Chillag, and Spitz (1984) have examined whether horizontal and vertical axis tracking tasks combine as integral or separable dimensions of dual-axis tracking. Eventually, these multidimensional approaches to movement analysis may provide a greater integration of knowledge about perceptual and motoric symmetries.

#### REFERENCES

- Asatryan, D. G., & Fel'dman, A. G. (1965). Functional tuning of nervous system with control of movement or maintenance of a steady posture—I. Mechanographic analysis of the work of the joint on execution of a postural task. *Biofizika*, *10*, 837-846.
- Barnes, G. R., & Sommerville, G. P. (1978). Visual target acquisition and tracking performance using a helmet-mounted sight. *Aviation, Space, and Environmental Medicine*, *49*, 565-572.
- Card, S. K., English, W. K., & Burr, B. J. (1978). Evaluation of mouse, rate-controlled joy stick, step keys, and text keys for text selection on a CRT. *Ergonomics*, *21*, 601-613.
- Coombs, C. H., Dawes, R. M., & Tversky, A. (1970). *Mathematical psychology*. Englewood Cliffs, NJ: Prentice-Hall.
- Corrigan, R. E., & Brogden, W. J. (1949). The trigonometric relationship of precision and angle of linear pursuit movements. *American Journal of Psychology*, *62*, 90-98.
- Crossman, E. R. F. W., & Goodeve, P. J. (1963). Feedback control of hand-movement and Fitts' Law. Paper presented at the meeting of the Experimental Psychology Society, Oxford, July 1963. Published in *Quarterly Journal of Experimental Psychology*, 1983, *35A*, 251-278.
- Draper, N., & Smith, H. (1966). *Applied Regression Analysis*. New York: Wiley.
- Drury, C. G. (1975). Application of Fitts' Law to foot-pedal design. *Human Factors*, *17*, 368-373.
- Falkenburg, L. E., & Newell, K. M. (1980). Relative contribution of movement time, amplitude, and velocity to response initiation. *Journal of Experimental Psychology*, *6*, 760-768.
- Fitts, P. M. (1947). A study of location discrimination ability. In P. M. Fitts (Ed.) *Psychological research on equipment design* (Research Report 19). Army Air Force, Aviation Psychology Program.
- Fitts, P. M. (1954). The information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, *42*, 381-391.

- Fitts, P. M., & Peterson, J. R. (1964). Information capacity of discrete motor responses. *Journal of Experimental Psychology*, 67, 103-112.
- Gibbs, C. B. (1962). Controller design: Interactions of controlling limbs, time-lags and gains in positional and velocity systems. *Ergonomics*, 5, 385-402.
- Hartzell, E. J., Dunbar, S., Beveridge, R., & Cortilla, R. (1982). Helicopter pilot response latency as a function of the spatial arrangement of instruments and controls. *Proceedings of the Eighteenth Annual Conference on Manual Control*, 345-364.
- Howarth, C. I., Beggs, W. D. A., & Bowden, J. M. (1971). The relationship between speed and accuracy of movement aimed at a target. *Acta Psychologica*, 35, 207-218.
- Jagacinski, R. J., Repperger, D. W., Moran, M. S., Ward, S. L., & Glass, B. (1980). Fitts' Law and the microstructure of rapid discrete movements. *Journal of Experimental Psychology: Human Perception and Performance*, 6, 309-320.
- Jagacinski, R. J., Repperger, D. W., Ward, S. L., & Morgan, M. S. (1980). A test of Fitts' Law with moving targets. *Human Factors*, 22, 225-233.
- Jorgeson, C. M. (1966). A study of the variables affecting human motor responses. Unpublished manuscript, University of Michigan.
- Keele, S. W. (1968). Movement control in skilled motor performance. *Psychological Bulletin*, 70, 387-403.
- Klapp, S. T. (1975). Feedback versus motor programming in the control of aimed movements. *Journal of Experimental Psychology: Human Perception and Performance*, 104, 147-153.
- Klapp, S. T. (1981). Motor programming is not the only process which can influence RT: Some thoughts on the Marteniuk and MacKenzie analysis. *Journal of Motor Behavior*, 13, 320-328.
- Langolf, G. D., Chaffin, D. B., & Foulke, J. A. (1976). An investigation of Fitts' Law using a wide range of movement amplitudes. *Journal of Motor Behavior*, 8, 113-128.
- Levison, W. H., Zacharias, G. L., Porterfield, J. L., Monk, D., & Arbak, C. (1981). A comparison of head and manual control for a position-control pursuit tracking task. *Proceedings of the Seventeenth Annual Conference on Manual Control*, 641-652.
- McCormick, E. J., & Sanders, M. S. (1982). *Human factors in engineering and design* (5th ed.). New York: McGraw-Hill.
- Meyer, D. E., Smith, J. E. K., & Wright, C. E. (1982). Models for the speed and accuracy of aimed limb movements. *Psychological Review*, 89, 449-482.
- Navon, D., Gopher, D., Chillag, N., & Spitz, G. (1984). On separability of and interference between tracking dimensions in dual-axis tracking. *Journal of Motor Behavior*, 16, 364-391.
- Pachella, R. G., Somers, P., & Hardzinski, M. (1981). A psychophysical approach to dimensional integrality. In D. J. Getty & H. Howard, Jr., (Eds.), *Auditory and visual pattern recognition*. Hillsdale, NJ: Erlbaum.
- Quinn, J. L., Jr., Schmidt, R. A., Zelaznik, H. N., Hawkins, B., & McFarquhar, R. (1980). Target-size influences on reaction time with movement time controlled. *Journal of Motor Behavior*, 12, 239-261.
- Schmidt, R. A., Zelaznik, H., Hawkins, B., Frank, J. S., & Quinn, J. T., Jr. (1979). Motor-output variability: A theory for the accuracy of rapid motor acts. *Psychological Review*, 86, 415-451.
- Shepard, R. N. (1964). Attention and the metric structure of the stimulus space. *Journal of Mathematical Psychology*, 1, 54-87.
- Siddall, G. J., Holding, D. H., & Draper, J. (1957). Errors of aim and extent in manual point to point movement. *Occupational Psychology*, 31, 185-195.
- Soede, M., Stassen, H. G., van Lunteren, A., & Luitse, W. J. (1973). A light spot operated typewriter for physically handicapped patients. *Ergonomics*, 16, 829-844.

Submitted September 9, 1983  
Revision submitted March 6, 1984