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SIMPLE REACTION TIME AS A FUNCTION OF TIME UNCERTAINTY[1]

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An earlier study (3) showed that simple reaction time (RT) varies with *S*'s uncertainty about time of stimulus occurrence. This time uncertainty is a function of both the mean duration of the time (foreperiod) between a warning signal and the stimulus and the variability within the series of foreperiods. Foreperiod variability adds uncertainty directly and mean foreperiod is important since *S*'s ability to predict time of stimulus occurrence is very much a function of the length of time he must predict.

In the previous report the two sources of time uncertainty were considered separately because no single measure was available. The present experiment illustrates with new data, a method by which all of *S*'s time uncertainty can be expressed as a single number and reaction time plotted as a single-valued function of time uncertainty. In addition, time uncertainty is expressed in terms of the information measure.

In order to estimate the amount of time uncertainty due to *S*'s imperfect time-keeping ability, it is necessary to run time-interval prediction tests with intervals equal to the mean foreperiods of the reaction time tests. The variance of the distribution of each *S*'s times of response in the time prediction test is taken as a measure of his "subjective" time uncertainty for intervals equal to the predicted interval. Total time uncertainty is obtained by adding this measure of subjective time uncertainty to the variance of the distribution of foreperiods in the RT test having a mean foreperiod equal to the time interval used in the prediction test. This total variance can be converted to a nondimensional informational measure of uncertainty which makes possible a comparison of the present results with RT tests involving a choice reaction.

The present study, then, consists of two separate tests series, one RT and one time prediction, given to the same *Ss*. The results of the two series are combined in such a way that a single valued plot of RT as a function of time uncertainty is derived for each *S*.

Method

RT apparatus.—The apparatus consisted of an NE 51 neon stimulus bulb, a response key, and a warning click device. The .02-sec. stimulus was clearly visible in the dimly lighted room. In another room a teletype tape programmer presented 11 different lengths of foreperiod in a random order as described below. The warning click occurred regularly every 10 sec. Each RT was measured to the nearest millisecond by a Berkeley Universal counter and timer, Model 5510, and printed out automatically.

RT procedure.—Ten different RT tests were used, each with a different mean foreperiod and/or foreperiod variability. These tests are described in Table 1. Each *S* took single runs on each of the 10 tests in reversing order 1 → 10, 10 → 1 until he had taken one practice run and five experimental runs on each test. Three *Ss* began with Test 10 and two began with Test 1. Each run consisted of 51 stimulus presentations.

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TABLE 1

DESCRIPTION OF REACTION TIME TESTS

Test	Foreperiod Characteristics (Sec.)		
	Mean	*SD*	Range
1	0.5	0	0
2	1	0	0
3	2	0	0
4	4	0	0
5	8	0	0
6	0.5	0.11	0.5
7	1	0.23	1
8	2	0.46	2
9	4	0.91	4
10	5	1.82	8

Note.—The variable foreperiods were chosen randomly from controlled frequencies in the shape of a normal distribution having a range of $\pm$ 2.2 *SD*.

The *S*s were always informed of the range of foreperiods before each run, and in addition, the first three trials in the test demonstrated the range. The first foreperiod of the test was always the longest for that test, the second foreperiod the shortest, and the third foreperiod the mean foreperiod for that test. These first three RT's were omitted from the analysis. The remaining 48 foreperiods were randomly ordered from the normal distribution of foreperiods described in Table 1. Note that this is a change from the previous study in which the frequency distribution of foreperiods was rectangular. In the constant foreperiod tests, 51 constant foreperiods were used in each run and the first 3 RT's were omitted from the analysis.

In all tests, *S* was instructed to respond as soon as possible after the stimulus light, but never before. If a response occurred before the stimulus in any run, the run was halted and started over. This method produced considerably less than 1% anticipations, none of which occur in the runs reported here.

Prediction apparatus.—The prediction apparatus used a warning click every 10 sec. and a response key similar to the RT apparatus. No stimulus light was used, however. Instead, a small light would flash at the instant *S* pressed the key. This light varied in position according to how long after the warning click the key was pressed. If the key was pressed at exactly the instructed interval after the click, the light would appear on an index mark; if pressed too soon, the light appeared to the left of this mark; if pressed too late, to the right. The distance in each case was proportional to the error and so *S* received immediate knowledge of direction and magnitude of error after each response. The interval between each click and prediction response was recorded from a Standard Electric timer.

Prediction procedure.—Tests were given using five prediction intervals: ½, 1, 2, 4, and 8 sec. Each *S* took single runs of 50 stimuli each on each test in reversing order ½ → 8, 8 → ½, until he had completed one practice and five experimental runs on each test. Three *S*s began with the 8-sec. test and two began with the ½-sec. test. The *S*s were always informed of the correct interval before each test and given at least four "warm-up" predictions followed without interruption by the 50 clicks, 10 sec. apart for the test run.

In all tests, *S* was instructed to make a prediction after each click and attempt to make the light appear as close as possible to the index mark.

Subjects.—The *S*s were two university students and three laboratory personnel. Subjects K, B, and C had previous RT training; subjects G and W did not. All *S*s took the RT tests before the time prediction tests.

RESULTS AND DISCUSSION

The total uncertainty of time of stimulus occurrence was computed for each *S* separately for each of the 10 RT tests. The assumption made is that *S*s uncertainty about the time of stimulus occurrence in any test may be estimated by adding the variance of the distribution of actual foreperiods to the variance of his own predictions of time intervals equal to the mean foreperiod. Reaction-time Tests 1–5 have no foreperiod variability so that all of the time uncertainty is accounted for by this estimate. Tests 6–10 have contributions from both foreperiod variability and *S*'s subjective time uncertainty. Test 10 uses a mean foreperiod of 5 sec. for which prediction data was not taken but the *SD* of prediction is a nearly linear function of prediction interval in this region, and so the variance for 5-sec. prediction is obtained by interpolating *SD*'s between 4 and 8 sec.

The total time uncertainties were converted to *SD*'s and these *SD*'s used

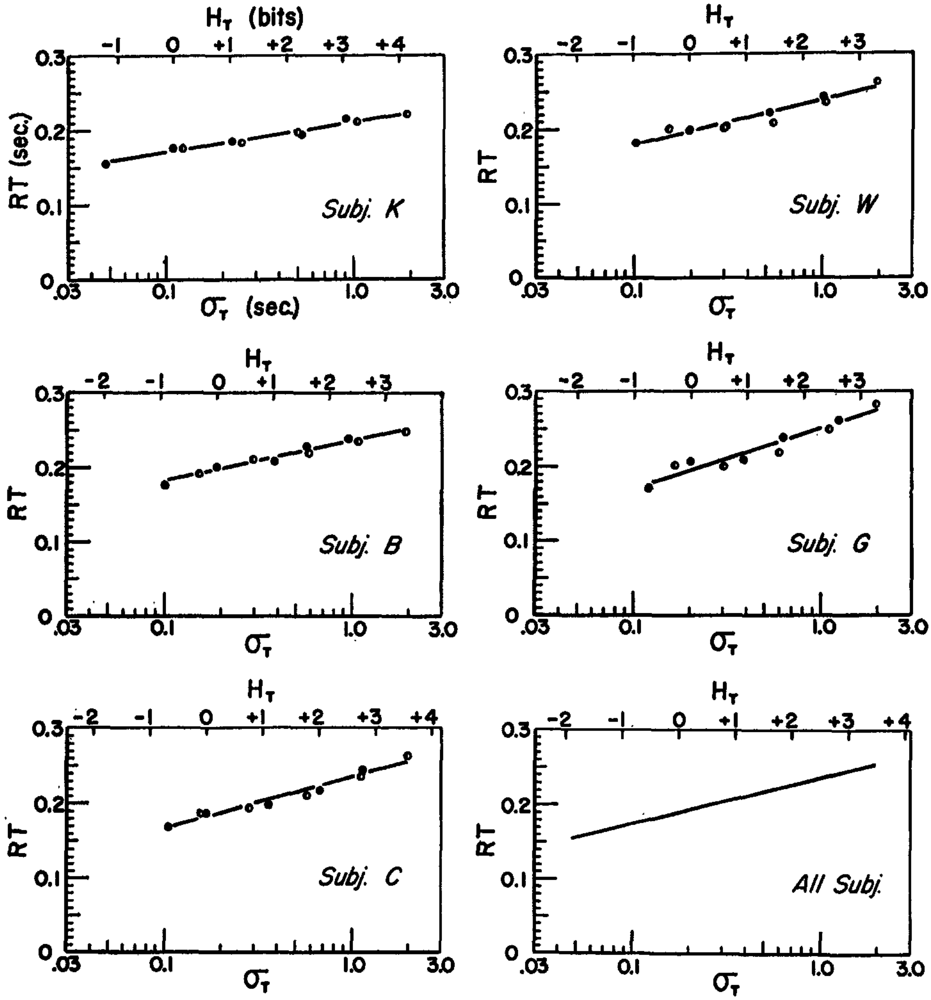

Fig. 1. Reaction time as a function of time uncertainty of the stimulus. σ_T is *S*'s time uncertainty, given as an *SD*, and found by adding foreperiod variance to the variance of *S*'s estimates of intervals equal to the mean foreperiod. H_T is time uncertainty in bits relative to a constant 1-sec. foreperiod. Straight lines are fitted by least squares.

as the time uncertainty dimension of the plots in Fig. 1. Time uncertainty (σ_T) appears on a log scale along the abscissa. Mean RT is plotted on a linear scale along the ordinate. The filled dots represent RT Tests 1–5, reading from left to right, and the open dots represent Tests 6–10, also reading from left to right. Each point represents the mean of 5 runs of 48 stimuli each.

Time uncertainty is expressed in terms of the informational measure (bits) along the upper ordinate scale of Fig. 1. Since time is a continuous variable, the stimulus uncertainty measure must be relative to some standard distribution[2] (2). Shan-

[2] Information transmission values are absolute rather than relative, even when based upon continuous distributions. Transmission is discussed later.

non's formulas (4) assume that the standard distribution is a uniform distribution over one unit of the variable, but for the present data it has seemed better to take as the standard each *S*'s own time uncertainty for a constant foreperiod of 1 sec. Thus the zero value of informational uncertainty is placed directly over the second filled dot corresponding to Test 2 which uses a constant foreperiod of 1 sec. The other points along the scale of presented information are found in the following manner. The *SD*, σ_T, which is taken as the measure of total time uncertainty is assumed to be based on a normal distribution since it is the result of adding variances from foreperiod and time-prediction distributions which are approximately normal in shape. The uncertainty in bits to be associated with a normally distributed random variable is given by Shannon (4).

$$H(x) = \log\sqrt{2\pi e}\,\sigma \qquad (1)$$

If the Shannon formula were used directly with σ_T in seconds, the uncertainty values obtained would be relative to a uniform distribution over 1 sec. In order to make the informational uncertainty measure relative to the distribution of estimates of a 1-sec. interval, it is necessary to use an arbitrary unit of time for measuring σ. These new time units will be a linear function of σ_T, so that we may write:

$$H_T = \log k\sigma_T + \log\sqrt{2\pi e} \qquad (2)$$

in which σ_T is still in seconds, but H_T is relative to the desired standard distribution if $H_T = 0$ when $\sigma_T = \sigma_1$, where σ_1 is the *SD* of *S*'s estimates of a 1-sec. interval. By substitution:

$$0 = \log k\sigma_1 + \log\sqrt{2\pi e}, \qquad (3)$$

$$H_T = \log\sigma_T - \log\sigma_1.$$

For uncertainty in bits relative to a constant 1-sec. foreperiod:

$$H_T = \log_2\sigma_T - \log_2\sigma_1. \qquad (4)$$

This equation is used to compute the upper abscissa scale in Fig. 1. The negative informational uncertainties to the left of zero represent less time uncertainty than the standard one-second foreperiod.

The straight lines in Fig. 1 are fitted to the points by the least mean square method. The line for the "All Subj." plot is fitted to the total array of points as plotted in the individual graphs. For the sake of clarity, the individual *S* points are not repeated in the combined plot. The zero point of the informational scale on the combined plot is based on the average variance of response times in the one-second prediction tests.

Product-moment correlation between mean RT and time uncertainty in bits (or in log σ_T) varies from .956 to .983 among the five *S*s. When the data from all *S*s are pooled, the correlation drops to .905 because of the large difference in slope of regression line among the *S*s. This slope varies from 12 msec. per bit to 24 msec. per bit over *S*s. The pooled data shows a slope of 18 msec. per bit with an equation in terms of time uncertainty given as an *SD* of:

$$\text{RT} = .018\log_{10}\sigma_T + .235, \qquad (5)$$

where $\sigma_T = \sigma_S^2 + \sigma_F^2$, σ_F^2 is foreperiod variance, and σ_S^2 is variance of *S*'s own estimates of interval equal to mean foreperiod, with all values in seconds.

An analysis of variance of the data from each *S* separately showed no significant variance due to deviations from linear regression between RT and time uncertainty. This finding, together with the high linear correlations, suggest that RT is a linear function of time uncertainty in the range of this study.

Several investigators have studied the relation between RT and stimulus uncertainty in situations in which there was uncertainty about which of several stimuli will be presented. In general, they have also found a linear relation between RT and stimulus uncertainty in bits, but the slopes have varied widely (1). No slope, however, has been as small as the 12–24 msec. per bit found in the present study. The difference in slope are due to such things as stimulus-response compatibility and dimensionality. More work needs to be done in these areas.

The next question that arises concerning time uncertainty in informational terms is how much information about time of stimulus occurrence *S* actually transmits.[3] Transmitted information may be approximated by the difference between *S*'s prestimulus uncertainty about when the stimulus will occur and the residual uncertainty about time of occurrence as estimated from his response. If we neglect the slight relation between foreperiod and RT, and assume normal distributions, this calculation involves only taking the logarithm of the ratio of the time uncertainty expressed as an *SD* (σ_T) and the *SD* of the corresponding RT distribution. The constant factor which makes the information measure relative, drops out in this ratio so that transmitted information is an absolute score. The equations for this approximation are given below.

As before, *S*'s average informational uncertainty about when each stimulus will occur is given by:

$$H_T = \log_2\sigma_T - \log_2\sigma_1. \quad (6)$$

By the method shown above, the average informational uncertainty about time of stimulus occurrence based upon knowledge of the response time is given by:

$$H_{RT} = \log_2\sigma_{RT} - \log_2\sigma_1. \quad (7)$$

Transmitted information is given by:

$$T = H_T - H_{RT} = \log_2\sigma_T/\sigma_{RT}. \quad (8)$$

The above method of measuring transmitted information is different from a straightforward application of the usual transmission formulas which would consider uncertainties in clock time only. For tests with considerable clock-time uncertainty in the stimulus, the two methods give essentially the same results, but for any test with constant foreperiod, the direct application of transmission formulas would show zero transmission. The inclusion of subjective time uncertainty in the present study gives a more accurate picture of the actual informational demands on the human operator.

In the present data σ_T varies over a 20-to-1 range over tests while the *SD* of the RT distributions varies within only a 2-to-1 range. This means that time uncertainty of the stimulus determines the slope of the RT versus information transmitted functions which, therefore, are very similar to the RT vs. log σ_T functions as plotted in Fig. 1. Of more interest is the absolute value of information transmitted. The peak transmission occurred in Test 10 and varies very little over *S*s: 5.37 to 5.60 bits per stimulus. The highest ratio between information transmission and RT results from a transmission of 5.49 bits with a RT of 222 msec. in Test 10. The lowest transmission occurs in Test 1 where *S* has little uncertainty about time of stimulus onset. The smallest ratio here involves a transmission of .86 bits with a 157-msec. RT. Interestingly, the lowest and highest ratios were both achieved by the same *S*.

Note that the information transmission values cannot be compared to the stimulus time uncertainty measured in bits. The transmission scores are absolute, but the stimulus uncertainty meas-

[3] Tests 1–5 present no information in clock time to be transmitted but do involve considerable subjective time uncertainty which is reduced by stimulus occurrence. Transmission has been measured in terms of this reduction in uncertainty.

ure is relative to an arbitrary standard distribution.

Summary

Five *S*s were given a set of simple RT tests specifically designed to test the hypothesis that a single-valued relation could be obtained between RT and the time uncertainty of the stimulus. This relation was shown to be approximately linear when time uncertainty is plotted as an informational measure. The slope of the RT-time uncertainty function averaged 18 msec. per bit of stimulus uncertainty which is less than the slope arising from RT experiments involving choice among several stimuli previously reported. Information transmitted in the time domain varied from less than one to more than five bits per stimulus over the 10 tests.

References

1. Bricker, P. D. Information measurement and reaction time: a review. Henry Quastler (Ed.). *Information theory in psychology*. Glencoe, Ill.: Free Press, 1955.
2. Klemmer, E. T. *The information content of polar coordinates*. Cambridge, Mass.: Air Force Cambridge Res. Center Tech. Rep. 54-54, 1954.
3. Klemmer, E. T. Time uncertainty in simple reaction time. *J. exp. Psychol.*, 1956, **51**, 179–184.
4. Shannon, C. E. A mathematical theory of communication. *Bell System Tech. J.*, 1948, **27**, 623–656.

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