Planning Reaches by Evaluating Stored Postures

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This article describes a theory of the computations underlying the selection of coordinated motion patterns, especially in reaching tasks. The central idea is that when a spatial target is selected as an object to be reached, stored postures are evaluated for the contributions they can make to the task. Weights are assigned to the stored postures, and a single target posture is found by taking a weighted sum of the stored postures. Movement is achieved by reducing the distance between the starting angle and target angle of each joint. The model explains compensation for reduced joint mobility, tool use, practice effects, performance errors, and aspects of movement kinematics. Extensions of the model can account for anticipation and coarticulation effects, movement through via points, and hierarchical control of series of movements.

The goal of this research is a unified theory of the planning and control of physical action. Such a theory, as several authors have noted (Jeannerod, in press; Rosenbaum, 1991; Wing, 1993), has been lacking. Instead, specialized models have been designed to account for data from different tasks. The sentiment underlying our work is that attempting to develop a unified theory forces one to confront issues that might otherwise be ignored. In addition, the enterprise helps one see common properties of apparently disparate systems.

The aim of the theory under development is to solve the inverse kinematics problem. Here one must find a set of joint angles (i.e., a posture) that allows one or more points on the limb-segment chain to occupy one or more desired spatial locations. The problem derives from the fact that an infinite number of postures usually satisfy this requirement. For example, in reaching for an object, there are typically infinitely many ways that the hand can be brought to the object.

The inverse kinematics problem is an instance of the broader degrees of freedom problem, first articulated by Bernstein (1967). The degrees of freedom problem arises when more variables exist than are strictly required to complete a task. The problem is not specific to motor control but arises in other contexts as well. In view of the generality of the problem, solutions to it within particular contexts, such as motor control, may extend to other domains.

Our attempt to solve the inverse kinematics problem is embodied in a computational model that makes three central claims. First, movements are selected on the basis of the activity of stored posture representations. Second, the stored posture representations are activated in relation to their fits to task demands. Finally, the output of the system as a whole is based on the pooling of these activations. The model is called Knowledge II. An account of its predecessor, now called Knowledge I, was published earlier (Rosenbaum, Engelbrecht, Bushe, & Loukopoulous, 1993a, 1993b). Our reason for using the term knowledge is that we believe skillful movement depends as much on stored information about the body and its interactions with the environment as it does on the physical makeup of the musculoskeletal system and the energy fields in which it behaves. Our commitment to the cognitive substrates of motor behavior follows analyses of alternative perspectives that seek to minimize
the role of cognitive representations (Kelso, 1981; Turvey, 1990a); these analyses appear in Jordan and Rosenbaum (1989), Pew and Rosenbaum (1988), Rosenbaum (1991), and Rosenbaum and Krist (in press). We believe that representations should not be adduced unnecessarily when trying to explain behavior but that they should be admitted if avoiding them limits unnecessarily what one can explain.

Knowledge II was designed to remedy several limitations of Knowledge I while preserving most of its core assumptions. In developing these two versions of the model, we have pursued a strategy of successive approximation. We have refined and elaborated our assumptions so that we can account for more data and generate new predictions. Our goal has been as much to provide a framework in which to pose new questions as to offer a crystallized explanation for previous results. In our attempt to develop such a framework, we have tried to make assumptions that are psychologically, neurologically, and physically plausible and to make theoretical claims that are as principled and parsimonious as possible.

Because we expect many readers to be unfamiliar with Knowledge I and to be more interested in the new improved model than its predecessor, we present Knowledge II alone, noting its differences from Knowledge I only when necessary. We discuss a large number of phenomena that Knowledge II can account for that Knowledge I could not account for or that were not noted in the earlier presentations of Knowledge I. When we present a result that can be accounted for by Knowledge I as well as Knowledge II, we state that the Knowledge model (or simply the model) can account for the result; however, when we present a result that can be accounted for only by Knowledge II, we refer to Knowledge II directly. There are no results that can be accounted for by Knowledge I but not Knowledge II.

Before we describe the model, we review the inverse kinematics problem and the degrees of freedom problem. Then we present Knowledge II as well as some alternative versions of it that we have considered. Next, we describe the phenomena the model can explain and extensions and elaborations of the model that can be pursued in the future. In the last part of the article, we compare our approach with other approaches to movement planning and control, and offer some final remarks about the promise of our general approach.

The Inverse Kinematics Problem

Consider Figure 1, which shows a simple stick figure that can bend at the hip, shoulder, and elbow. Because the stick figure can move about three mechanical axes, it has 3 degrees of freedom, and because bending at the hip, shoulder, and elbow permits the stick figure to move forward and back and up and down, its motion is restricted to a plane in extrinsic space. Any point in a plane can be defined by two values, its x and y values (assuming a Cartesian coordinate system) or its distance from a reference point and angle with respect to a reference line (assuming a polar coordinate system). Because the posture adopted by the stick figure is defined by three values (the hip, shoulder, and elbow angles), more degrees of freedom characterize its posture than characterize an external point to which the stick figure must reach. As a result, spatial targets within the stick figure's work space (i.e., within the spatial region that the stick figure can reach) can be arrived at with infinitely many postures (although spatial targets at the edge of the stick figure's work space can be reached with only one posture). For spatial targets within the
work space, the mapping of spatial targets to postures is therefore one to many, which means that the problem of finding a posture that permits the hand to contact such a spatial target is mathematically ill posed. By extension, finding a movement to the target is ill posed as well, because a movement can be viewed as a sequence of postures. Unless one selects movements randomly—

a possibility that flies in the face of everyday experience—other constraints must be posited to explain how reaches are selected.

The problem just described is called the inverse kinematics problem. The term refers to the fact that in mapping back (hence inverse) from a spatial target to a posture, the mapping is one to many. The problem involves kinematics because it pertains to positions without regard to the forces that yield them. When forces must be found that give desired positions, the problem is called inverse dynamics (Craig, 1986). The forward dynamics problem arises when positions must be found given known forces. The forward kinematics problem arises when positions must be found given known joint angles.

As stated earlier, the inverse kinematics problem is an instance of the broader degrees of freedom problem (Bernstein, 1967), which has been approached in three ways in motor-control research. One way, originating with Bernstein (1967), is to posit interactions between or among effectors. The idea is that such interactions can reduce the degrees of freedom that must be considered. For example, if the elbow and shoulder are dependent, the degrees of freedom problem can be eliminated for a creature that bends only at the hip, shoulder, and elbow to reach a point in a plane. Interactions between effectors can be dramatic, as anyone who has tried to draw a circle with one hand while drawing a square with the other can attest. Many studies have confirmed the strength and regularity of effector interactions (Heuer, in press; Turvey, 1990b).

The second way that the degrees of freedom problem has been approached is mechanical. The idea is to exploit physical properties of the musculoskeletal system and its interactions with the environment. An example is reliance on gravity during locomotion. Muscle activity in the leg is largely unnecessary during the swing phase of locomotion, when the leg pivots forward (McMahon, 1984). The leg literally drops during the swing phase, making it unnecessary to plan an explicit movement trajectory or issue motor commands to the leg muscles during this phase. Similar examples have been reported in other domains (Bizzi & Mussa-Ivaldi, 1989; Hasan & Stuart, 1988; Rosenbaum & Krist, in press; Thelen, Kelso, & Fogel, 1987), suggesting that reliance on mechanics can be an effective method for simplifying the control of the degrees of freedom relevant to effective motion.

The third type of solution to the degrees of freedom problem is based on cost containment. A reason for pursuing this strategy is that reliance on interactions and mechanics is not guaranteed to eliminate redundant solutions. As a simple example, it is possible to grasp a glass with one's left hand or right. Normally, neither mechanics nor effector interactions determine which hand should be used. Nevertheless, the choice is nonrandom: One normally does not reach across one's body to pick up the glass, although one could if necessary (e.g., if one were holding a briefcase with the other hand). This outcome suggests that costs are taken into account in deciding which hand to use or, more generally, in determining which action to perform. That cost evaluation plays a role in movement selection is further reflected in the fact that movements that are awkward in some contexts are inappropriate in other contexts. It may be awkward to touch one's left ear with one's right hand by snaking one's right hand around the back of one's head, for example, but this is the preferred mode of action if one is hanging from the edge of a cliff with one's left hand and with one's face up against a rock. As this example is meant to show, the degrees of freedom problem is not really a problem in the sense that it would be better if it did not exist. Having a plethora of behavioral options provides one with a wide range of behaviors that can be deployed as task demands warrant. The degrees of freedom problem is a problem only insofar as students of behavior need to understand how particular behaviors are found when alternative behaviors are possible. The Knowledge model was designed for this purpose.

Knowledge II

The specific task that the Knowledge model was designed to perform is one-handed reaching in the sagittal plane extending through the shoulder of the reaching hand. One-handed sagittal-plane reaching is a convenient task to model because it can involve the whole body (from the toes to the fingers) and because it permits relatively easy attainment of behavioral data to test predictions. The theory does not yet address two-handed reaching or other kinds of motor behavior, and it has not yet been applied to movement in three-dimensional space. The model is also purely kinematic at this stage; it ignores forces acting on and imparted by the actor, so it does not confront the inverse dynamics problem. Despite these limitations, the model has been designed to extend to other motor tasks and effector subsystems, to extend to motions in three extrinsic spatial dimensions, and to include dynamics as well as kinematics. In fact, considering the model's claims at an abstract level, it may even extend to nonmotor systems.

Overview of the Model

The model works as follows. The actor first identifies a spatial target to be reached, relying on perceptual, attentional, and decision processes to do so. Planning a movement is based on an evaluation of stored posture representations. The idea that postures are the stored instances used for motor planning is the key new hypothesis of the model. It is possible to think of the process of evaluating stored postures in terms of posture "demons" making bids, potentially in parallel, to a higher level demon about their suitability for the task. When all of the bids have been collected, a single target posture is found, and then a movement trajectory is created from the starting posture to the target posture.

The mathematical ideas of the model are schematized in Fig-

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1 Sagittal-plane reaching in an upright posture is useful to study from a dynamics perspective because gravitational and other forces are likely to play an important role in such behavior (Barin, 1989), making the limitations of a purely kinematic approach apparent.
Figure 2. Schematic overview of the knowledge model. Postures are represented as vectors in joint space. The dimensions of the space shown here are hip, shoulder, and elbow angles. Panel A: At the start of a reaching task, there is a starting posture and, in this case, three stored postures: $P_1$, $P_2$, and $P_3$. Panel B: After a spatial target is identified, weights $w_1$, $w_2$, and $w_3$ are assigned to the three posture vectors. For this reaching task, $P_3$ is assigned the largest weight (it is scaled down least), whereas $P_1$ is assigned the smallest weight (it is scaled down most). Note that the spatial target to be reached for is not shown because a spatial target is represented in extrinsic spatial coordinates. Panel C: The stored postures are summed vectorially, yielding a target posture. Panel D: A trajectory is created from the starting posture to the target posture. The intermediate postures, denoted as dashed vectors, are spaced more closely together at the start and end of the movement because the movement of each joint is assumed to have a bell-shaped angular velocity profile.

As shown in Figure 2A, postures are represented as vectors, the dimensions of which are the mechanical degrees of freedom of the motor system (or the subset of mechanical degrees of freedom being modeled). In the example shown in Figure 2A, the dimensions are hip, shoulder, and elbow angles, variations of which produce displacements in the sagittal plane.

When a spatial target is identified, the stored postures are assigned weights based on their judged effectiveness for the reaching task (see Figure 2B). The greater the judged effectiveness of a posture, the larger the weight it receives. The weights can take on values between 0 and 1 and must sum to 1. The criteria for assigning weights to stored postures reflect spatial accuracy costs (the extent to which the stored postures miss the target) and travel costs (how expensive it will be to move to the stored postures from the starting posture). Because both types of costs are taken into account, if a stored posture achieves the spatial target, it will receive a weight that approaches 1 as its travel cost approaches 0. If a stored posture fails to achieve the spatial target, it may nonetheless receive more weight than a stored posture that achieves the spatial target if it has a significantly lower travel cost.
Because the weights can take on values between 0 and 1, when the weights are assigned to the stored postures, the vectors corresponding to the stored postures are scaled down by an amount that depends on their judged effectiveness for the task. Vectors corresponding to stored postures that are judged highly effective for the task receive large weights and so are scaled down little, whereas vectors corresponding to stored postures that are judged less effective receive smaller weights and so are scaled down more. All stored-posture vectors are weighted and so, as a result, take on lengths that range from 100% of their original lengths (if their associated weight is 1) down to a length approaching 0% of their original length (as their associated weight approaches 0). Only stored postures are weighted. Thus, the posture at the start of the task—the starting posture—is weighted only if it happens to be a stored posture.

After all of the stored postures have been scaled down, they are summed up (see Figure 2C). The result is a target posture. The summing of postures occurs in the same way that vectors are summed in any context—by joining the vectors head to tail, in any order, until all of them have been connected. The starting posture participates in the summing only if it is a stored posture.

The final step is creating a movement trajectory (see Figure 2D). This is achieved by allowing each degree of freedom to change continuously from its starting value (its value in the starting posture) to its target value (its value in the target posture). In simulating the kinematics of observed movements, the transition of each degree of freedom is assumed to follow a bell-shaped velocity profile. Normally, but not necessarily, the entire trajectory is assumed to describe a straight-line motion through joint space. This choice of trajectory is based on the expectation that biological action generally follows the principle of least action, which is a basic law in physics, best illustrated by the tendency of light to travel in straight lines. In our model, movements involving several joints generally start and end together to ensure least-action trajectories through joint space.

More details about the model are given below.

Storage

We have several reasons for assuming that stored postures provide the basis for movement selection and that stored movements do not (see Berthier, Singh, Barto, & Houk, 1993, for a movement-based model). Before turning to these reasons, we note that others before us have considered postures as motor planning units. Arbib, Iberall, and Lyons (1985) emphasized the role of hand-grip choices in the planning of reaching and grasping, and Klatzky and her colleagues (Klatzky, McCloskey, Doherty, Pellegrino, & Smith, 1987) distinguished four basic hand shapes for describing hand positions. Klatzky’s work followed a tradition begun by other investigators, both for postures of the hand (Malek, 1981; Napier, 1956) and of the body in general (Hewes, 1957). Although all of these authors have paid attention to postures, none of them has argued, as we do here, that postures are the primary units for movement planning.

Our first reason for favoring posture-based storage is that the dimensionality of posture space is clear, whereas the dimensionality of movement space is unclear. The dimensions of posture space are mechanical degrees of freedom (typically of the joints), whereas the dimensions of movement space can include any of a large number of possible features—starting and ending positions, mean velocities, peak velocities, mean accelerations, and so on. We believe that greater progress can be made modeling a system of known dimensionality than a system of unknown dimensionality.

Our second reason for favoring a posture-based system is that the mass-spring model of movement (Bizzi, Hogan, Mussa-Ivaldi, & Giszter, 1992; Crossman & Goodeve, 1983; Feldman, 1966; Kelso & Holt, 1980), which is one of the most influential models of motor control, suggests that the motor system “cares more” about the achievement of final positions (final postures) than about their means of achievement. In the mass-spring model, new limb positions are specified by setting the stiffnesses or restoring lengths of opposing muscles such that the new limb position is one for which the forces of the opposing muscles balance out. A key feature of the mass-spring model is that the full trajectory to the new limb position need not be specified explicitly; instead, it can come “for free” as a result of the dynamics of the musculoskeletal system. Setting new limb positions (new joint angles) by relying on the mass-spring properties of muscles does not preclude specification of the details of forthcoming trajectories by the motor system (for a review, see Rosenbaum & Krist, in press), but even if these details are specified, they can be computed as necessary with stored postures. It is consistent with the mass-spring model that postures are the main stored elements for motor control, because postures can be viewed as equilibrium positions in the mass-spring sense.

Our third reason for favoring a posture-based system is closely related to the second. When target muscle lengths are specified in a mass-spring control system, joint angles are implied. It is also true, but less well known, that when a target posture (a complete set of joint angles) is specified, a unique set of muscle lengths is implied (G. Caldwell, personal communication, November 1993; Shadmehr, 1993). Hence, the direction of determination between postures and muscle lengths is two way: Muscle lengths imply postures, and postures imply muscle lengths. This makes a posture-based system attractive because, when one hypothesizes target postures as goals for movement, the means of muscularly implementing those goals is not left for others. Note, however, that when muscle lengths are specified, muscle forces still need to be determined (unless one is at an extreme of the force–length curve for muscle). Thus, selecting postures does not solve the problem of defining muscle forces. We consider the specification of muscle forces later.

Our fourth reason for favoring a posture-based system is that research on memory for movements indicates that people recall limb positions better than limb movements (see Smyth, 1984, for a review). For example, as first shown by Laabs (1973) and then replicated by others, participants recall locations to which they bring the hand better than distances covered by the hand, even if the test movement begins from a different starting point than the original movement. This outcome corroborates the primacy of postural information over movement information.

Our fifth and final reason for favoring posture-based planning is that psychophysical ratings of postural comfort better ac-
count for movement choices than do psychophysical ratings of movement comfort (Rosenbaum, Vaughan, Jorgensen, Barnes, & Stewart, 1993). This result was obtained in studies of the end-state comfort effect (Rosenbaum et al., 1990): When participants reached for an object to be carried to another location, they typically grabbed the object in a way that maximized the comfort at the end of the transport task rather than at the beginning. Participants based their decisions about how to grab the object on considerations of postural comfort rather than on considerations of movement comfort, judging from psychophysical ratings of the postures that could be adopted at the end of the task in comparison with psychophysical ratings of the movements that could be used.

Now that we have reviewed our reasons for favoring posture-based storage, we state the assumptions we make about the storage system. First, the maximum number of posture representations that can be stored by an individual is assumed to be fixed, although the number can vary among individuals. Second, the distribution of posture representations can change with learning; the way it does so is described later in the Learning section. Third, each stored posture is assumed, under normal circumstances, to obey anatomical constraints. For modeling purposes, each joint occupies a fixed range of angles, and each limb segment has a length appropriate to the age and sex of the modeled actor; these values come from anthropometric tables or from observations. The fourth assumption is that stored postures can be lost as a result of accident or disease.

Planning

We now turn to the way the stored postures are used in planning. The goal of planning is to find a target posture. As stated earlier, when a spatial location is chosen as a target, the stored postures are evaluated for the contribution they can make to the task. The evaluations are made by assigning two costs to each stored posture—a spatial error cost and a travel cost. A weighted sum of these two costs is taken to arrive at a total cost for each stored posture. On the basis of the total costs of all of the stored postures, weights are assigned to the stored postures so that a single weighted sum—the target posture—can be found. Details follow.

Spatial error cost. The spatial error cost, \( S_p \), of stored posture \( p \) is the distance between the spatial target and the point along the effector chain (usually the hand) to be brought to the spatial target. We call this location along the effector chain the contact point. The contact point can be any point along the body or along a physical extension of the body (e.g., a hand-held tool).

For convenience, we code space in Cartesian coordinates and represent the distance between the spatial target, \( u \), and the contact point, \( v \), in terms of Euclidean distance:

\[
S_p = \{ (x_u - x_v)^2 + (y_u - y_v)^2 \}^{1/2}.
\]  

This definition of spatial error cost differs somewhat from the one used in Knowledge I, where we squared the Euclidean distance. Squaring had the effect of making the spatial error cost grow at a higher rate the farther the contact point was from the spatial target. Even so, the accuracy achieved was not always satisfactory. In Knowledge II, we introduce a new operation, "feedback" correction, that permits high accuracy even if the Euclidean distance is not squared. Feedforward correction is described later.

Travel cost. The second cost assigned to each stored posture is the travel cost. This is the estimated cost of moving from the starting posture to the stored posture under consideration. We define the travel cost as the sum of the costs of the movements of the individual joints (or, more properly, the mechanical degrees of freedom) in going from the starting posture to the stored posture. (For ease of exposition, we use the terms joint and degree of freedom interchangeably from this point onward. The usage is for convenience only, because a joint can have more than 1 degree of freedom. For example, the shoulder joint has 3 degrees of freedom.)

The definition of travel cost is based on the idea that if the actor were to adopt the \( p \)th stored posture given his or her starting posture, such that each of the \( j = 1, 2, \ldots, n \) joints moved through an absolute angular displacement of \( \alpha_j \) degrees in some amount of time \( T_j \), a cost, \( V_j(\alpha_j, T_j) \), would be incurred by each joint as it completed its movement, and the travel cost, \( V_p \), for the posture as a whole would be the sum of these costs:

\[
V_p = \sum_{j=1}^{n} V_j(\alpha_j, T_j).
\]  

Note that this definition ignores direction of movement and the portion of the joint's range of motion within which displacement occurs. Both factors are likely to be important when dynamics are taken into account. The definition also ignores interactions between the joints. Joint interactions are known to be important in actual performance, both at the level of neural coupling (Kots & Syrovoygin, 1966) and at the level of mechanical interaction (Hollerbach, 1990). We ignore these factors only to see how far we can go without considering them.

To develop a formula for the travel cost, \( V_j \), of the \( j \)th joint, we assume that there is an optimal time, \( T^*(\alpha_j) \), for the joint to cover an angular displacement \( \alpha_j \). We assume that this optimal time is a logarithmic function of \( \alpha_j \):

\[
T^*(\alpha_j) = k_1 \ln (\alpha_j + 1), k_2 \geq 0.
\]  

The value of 1 is added to \( \alpha_j \) because the natural logarithm of zero is undefined and because the logarithm of any positive number less than 1 is negative, and negative time is undefined here. We express \( T^*(\alpha_j) \) as a logarithmic function of \( \alpha_j \) for two reasons. First, the logarithmic function implies that \( T^*(\alpha_j) \) is a negatively accelerated function of \( \alpha_j \), which accords with the observation that mean limb velocity increases with limb placement (for a review, see Rosenbaum & Krist, in press). Second, Equation 3 is a special case of Fitts' law (Fitts, 1954) in that tolerance (the denominator in Fitts' law) equals 1. Equation 3 is also a generalization of Fitts' law in that Fitts' law typically applies to movements of an end effector (e.g., the hand) rather than to motions of individual joints. Because Fitts' law is a powerful predictor of movement times, it is an excellent candidate for a function relating the optimal time of a joint's motion to the angular displacement the joint must cover, although other functions are also possible (e.g., a square root
function). Which function is best must be determined empirically. A final comment about Equation 3 is that it allows $k$ to take on different values for different joints. The larger the value of $k$, the more the optimal time increases with angular displacement. We refer to $k$ as the joint's expense. A joint with a large $k$ value has a higher expense than a joint with a small $k$ value.

We next define the cost $V_j(a_j, T_j)$ of moving joint $j$ through an angle of size $a_j$ in a time $T_j$ that may or may not equal the joint's optimal time, $T_j^\star(a_j)$, for that same angular displacement:

$$V_j(a_j, T_j) = \frac{k_j a_j}{r} \left( 1 + \frac{T_j - T_j^\star(a_j)}{s^2} \right)^2,$$

(4)

where $r$ denotes the unit of angular displacement (in this case, 1 degree) and $s$ denotes the unit of time (in this case, 1 ms). Both terms are introduced to make the expression dimensionless and to specify the scale of the behavior being modeled. We square the difference between the proposed and optimal time so that the value of $V_j(a_j, T_j)$ grows with the difference between the times, regardless of the sign of the difference. We square the difference rather than take the absolute value for mathematical reasons only (to facilitate differentiation, as discussed later). Note finally that Equation 4, in combination with Equation 3, implies that $V_j$ is a cubic function of $k_j$.

Equation 4 implies a number of properties of $V_j(a_j, T_j)$, some of which are shown in Figure 3. First, $V_j(a_j, T_j)$ is zero when $a_j$ is zero, no matter what the value of $T_j$. Second, $V_j(a_j, T_j)$ is larger for joints with larger values of $k_j$. Third, $V_j(a_j, T_j)$ is larger as $a_j$ increases. Fourth, $V_j(a_j, T_j)$ is larger as the proposed time, $T_j$, deviates more and more from the optimal time, $T_j^\star(a_j)$. Finally, $V_j(a_j, T_j)$ grows with the squared difference between the proposed and optimal time; the larger the distance to be covered and the larger the joint's expense, the higher the rate of growth. Note that the last two properties are consistent with the fact that individual limb segments have resonant frequencies, that is, optimal periods (optimal frequency reciprocals) for required amplitudes (Fenn, 1938; Rosenbaum, Slotta, Vaughan, & Plamondon, 1991).

This formulation of travel cost allows movement times to be specified in different ways: (a) by allowing each joint to have its own optimal time, (b) by imposing the times externally, or (c) by requiring all joints to have the same movement time but letting the motor system pick that time on its own. We allow for all three methods because too few data are available yet to rule out any; however, as we said earlier, our preference is to assume a common movement time for all joints (ensuring a straight-line motion through joint space).

Consider each of these timing methods. The first one is to allow each joint to have its own optimal time, $T_j^\star(a_j)$, given the angular displacement it must achieve. This strategy minimizes the joints' costs of movement but leaves open the question of how the joints' movements are coordinated when their times differ. Few published data exist to indicate how this coordination might occur (e.g., whether the joints start moving together, stop moving together, or behave in some other way), although one study (Kaminski & Gentile, 1986) suggested that shoulder and elbow movements end together even if they begin at different times; this study concerned horizontal planar movement.

The second way that the joints' movement times can be specified is to impose the times externally. This can be done by an experimenter or coach giving instructions or feedback about times of movements. In the context of a computer simulation, it can be achieved by typing in desired values. Regardless of the source of the imposed times, each joint will then have a cost given the time and absolute angular distance it must cover, as specified in Equation 4. The travel cost for the posture as a whole will be the sum of these costs, as specified in Equation 2.

The third way that the joints' movement times can be specified is to let the planning system pick a common movement time for all the joints. We prefer this method because it satisfies the principle of least action for motion through joint space, because it does not beg the question of how times are specified when no coach or supervisor is available to supply times to the joints, and because it allows one to proceed without making ad hoc assumptions about how to coordinate the initiations or terminations of the joints' movements when the times of those movements differ.

How can the common, self-selected time be found? Suppose the best common movement time for a stored posture is the one that minimizes the stored posture's travel cost, $V_p$. To find that time, which we denote $T_p$, we treat $T_p$ as a variable and differentiate $V_p$ with respect to $T_p$. Then, setting $dV_p/dT_p = 0$, the value of $T_p$ that minimizes $V_p$ is

$$T_p = \frac{\sum_j k_j a_j T_j^\star(a_j)}{\sum_j k_j a_j}.$$

(5)

Once $T_p$ is found, it replaces $T_j$ in Equation 4 for all joints $j$, and the cost, $V_j(a_j, T_j)$, is found accordingly. Finally, the travel cost, $V_p$, for the entire posture is determined according to Equation 2.

Our intent in allowing for these three timing strategies is to open the way for studying movement timing patterns in a principled fashion based on alternative predictions arising from the Knowledge model.2 As we stated earlier, too few empirical data exist to allow us to choose only one timing strategy at this stage of research, and we think it is reasonable to hypothesize that the alternative methods may be available to biological actors depending on the tasks they must perform. Still, the method we favor, and the one we use in the simulations presented later in this article, is for the motor system to find a least costly common movement time. This common movement time can be used to permit all the joints to start and end their movements together.

Forward kinematics. Let us return to the first cost assigned to each stored posture, the spatial error cost. Recall that we defined the spatial error cost for a stored posture as the Euclidean distance

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2 This treatment of travel cost is far more complete in Knowledge II than in Knowledge I. In Knowledge I, the travel cost of a posture was treated as the sum of the joints' angular velocities; each joint's angular velocity was scaled by a term functionally equivalent to $k_j$. A problem with this formulation was that travel cost decreased monotonically with increases in movement time, which contradicts what is known about the resonance properties of limb segments (e.g., Fenn, 1938; Rosenbaum, Slotta, Vaughan, & Plamondon, 1991).
between the stored posture's contact point and the spatial target. To calculate the spatial error cost of a posture, one must know where the contact point will be in extrinsic space if the posture is adopted. In Knowledge II, this location is derived through forward kinematics. We assume that the nervous system can use trigonometry (or some neural analog thereof) to assign extrinsic spatial coordinates to any point of interest along the limb-segment chain, based on the lengths of the segments and the angles of the joints. This method differs from the one used in Knowledge I. There we assumed a lookup table indexed by spatial target locations; the entries in the table were postures that the hand reached to the spatial target location. There were several problems with this approach, however. First, it was unclear how many postures should be assigned to each spatial target location. Second, it was unclear which spatial target locations should be indexed. Third, the information from the table was useful only if the contact point was the hand; if some other part of the body was the contact point, it was unclear how to determine its spatial error cost. One might suppose that a lookup table could be constructed so that more than one part of the body is indexed by its spatial location, but this would leave open the question of which body parts are indexed, how many body parts are indexed, and how extensions of the body (tools) are indexed. Interpolation of spatial target indices might suffice to locate points along the body lying between indexed body sites, but extrapolation (forward kinematics) would be needed to locate points beyond them.

To compute forward kinematics, one needs a way of defining joint angles (see Figure 4). Following the convention used in

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**Figure 3.** Costs for a joint to cover angular distances of 15° or 45° in times ranging from 1 to 1,000 ms when the joint has an expense of $k = 111$ or $k = 167$. Cost is shown in linear coordinates in the left panel and, to bring out differences in costs around the minimum of each curve, in logarithmic coordinates in the right panel. The optimal times for the 15° and 45° displacements are 301 ms and 423 ms, respectively, when the joint's expense is $k = 111$, and 452 ms and 636 ms, respectively, when the joint's expense is $k = 167$. 

<table>
<thead>
<tr>
<th>$k$</th>
<th>Distance</th>
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<tr>
<td>111</td>
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Figure 4. Conventions used in defining joint angles.

robotics (Craig, 1986), we define a joint angle between limb segments $i$ and $i+1$ as the counterclockwise rotation that the linear extension of segment $i$ must go through in the sagittal plane to be aligned with segment $i+1$. The convention we adopt is that $i$ increases as one moves anatomically upward from the toe. Assume that the Cartesian location of the first joint is known; then, for limb lengths $l_1, l_2, \ldots, l_n$ originating at joint angles $\theta_1, \theta_2, \ldots, \theta_n$ respectively, the Cartesian location $(x_j, y_j)$ of joint $j$ is defined as follows:

$$
x_j = x_{j-1} + l_{j-1}\cos \sum_{i=1}^{j-1} \theta_i
$$

$$
y_j = y_{j-1} + l_{j-1}\sin \sum_{i=1}^{j-1} \theta_i.
$$

The capacity to compute forward kinematics has great utility for the actor. It makes it possible for him or her to aim for a spatial target with any part of the body, not just the hand. It is common knowledge that one can flip a switch with the shoulder when the arms are filled with groceries or nudge one's neighbor with one's elbow in a lecture hall. The capacity to compute forward kinematics makes it possible for the actor to achieve these simple tasks and also to aim for a spatial target with a tool, provided the length of the tool and its angle with respect to the body are known. It has long been believed that a hand-held tool effectively becomes an extension of the body (Gibson, 1979; Polanyi, 1964), and this belief has been reinforced by the ease with which animals and people can use tools for exploration (e.g., Kohler, 1925), even without visual feedback (Solomon & Turvey, 1988).

An aspect of personal experience supports the assumption that forward kinematics can be calculated for any point along the limb-segment chain. Consider a person bitten on one arm by a mosquito. Without thinking about it, and without looking, the person immediately slaps the bitten site with the hand of the other arm. To do this, the actor must have information about where on his or her skin the bite occurred. More important, the slapping hand moves to the place in extrinsic space where the bite occurred (i.e., the spatial location of the bitten site in relation to the spatial location of the slapping hand). This behavior suggests that people have the capacity to determine quickly and accurately where any point along the limb-segment chain is located in extrinsic space. Although this anecdote does not prove that forward kinematics can be computed, it suggests that it can be.

Computing the total cost. Once the spatial error cost and travel cost have been determined for the $p$th stored posture, the two costs are combined to yield a total cost, $C_p$, for that posture. We allow for the assignment of weights to the spatial error cost and to the travel cost depending on the relative importance of minimizing spatial errors versus travel costs:

$$
C_p = \frac{S_p}{\text{Max}S} + \frac{V_p}{\text{Max}V}.
$$

(7)

Here $w_s$ and $w_v$ denote the weights for minimizing spatial error cost and travel cost, respectively. Because $w_s$ and $w_v$ are nonnegative reals that sum to 1, only one weight needs to be specified explicitly. Max $S$ is the largest spatial error cost of any stored posture for the immediate reaching task, and Max $V$ is the largest travel cost of any stored posture for the immediate reaching task.\(^3\) The rationale for having a spatial error weight and a travel cost weight is that some tasks demand high spatial accuracy (performing brain surgery), whereas others do not (raising one's hand in class or doing jumping jacks); equal weighting, which is implied when there are no explicit weighting factors, is simply a special case.

Taking a weighted sum. Once the total cost has been obtained for all the stored postures, a single target posture is found by taking their weighted sum. It is possible to take a weighted sum of stored postures by treating the postures as vectors, the components of which are the joints' mechanical degrees of freedom. The weights assigned to the stored postures are based on their total costs.

We have three reasons for taking a weighted sum of the postures. First, this method makes it possible to find new postures. It is essential to generate novel motor behaviors for adaptive performance in everyday life (Schmidt, 1988). If the stored posture with the lowest total cost were always made the target posture, it would be impossible to generate target postures that were not part of the posture store.

Second, weighted summing appears to be used by the nervous system. The best-known example is neural population coding, used widely in the perceptual system (Erickson, 1984) and in

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3 Knowledge I did not include normalization. Normalization proved useful, however, when we worked with computer simulations of Knowledge II. Without it, we found it difficult to set coefficients when they occupied unbounded ranges.
the motor system (Georgopoulos, 1991). Weighted summing is not the only means by which decisions are made in the nervous system. Sometimes, decisions are made through other means, such as the winner-take-all strategy (C. D. Saltzman & Newcombe, 1994). In our approach, the winner-take-all strategy (i.e., making the least costly stored posture the target posture) can occur when one stored posture is assigned all the weight. However, this is just a special case of weighted summing, as shown later.

Our third reason for taking a weighted sum is that weighted summing helps the motor system perform robustly if posture representations are lost through accident or disease; the movements that are produced do not necessarily decrease in amplitude, as would occur if target postures were arrived at by taking simple, unweighted sums. Shorter-than-normal movements do characterize some motor disorders, but undershooting is not an essential feature of motorically expressed brain damage (David N. Levine, personal communication, March 12, 1992; Rothwell, 1987). Evidence consistent with weighted summing but inconsistent with simple, unweighted summing has been reported by Lee, Rohrer, and Sparks (1988) for the neural control of saccades.

To take a weighted sum of the stored postures, one must find weights. We call the method used for finding the weights Gaussian averaging. The method is based on the fact that weights assigned to stored postures must be inversely related to the stored postures' total costs. To transform total costs to weights, we first pass the total costs of the stored postures through a Gaussian filter. The filter yields a maximum output when the input total cost is zero (see Figure 5). The Gaussian requires a standard deviation, $\sigma$, which is set to the smallest total cost of any stored posture for the task being performed (i.e., the current reaching task). The advantage of setting $\sigma$ to the minimum total cost is that a stored posture with an intermediate total cost will be assigned a high Gaussian value when the smallest total cost of any stored posture is large (i.e., when no posture is especially well suited for the task), but the same stored posture will be assigned a low Gaussian value if the smallest total cost is small (i.e., when at least one stored posture is well suited for the task). This method prevents "too many cooks from spoiling the broth" when at least one cook is highly qualified for the job. By the same token, if a stored posture has a very small total cost, it will be assigned a very high Gaussian value and so will receive virtually all the weight. In the limit, when a stored posture's total cost is zero, it will receive all the weight and so will be anointed the target posture; this is the special case of weighted summing that amounts to a winner-take-all strategy.

Expressed mathematically, the filtering process entails passing the total cost $C_p$ for the $p$th posture through the Gaussian,

$$ G(C_p) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(C_p - \mu)^2}{2\sigma^2} \right], $$

where $\mu = 0$ and $\sigma = \min(C_p)$. Once all the stored postures have received Gaussian values, the weight, $g_p$, for the $p$th posture is set to the ratio of that posture's Gaussian value divided by the sum of the Gaussian values for all of the $p = 1, 2, \ldots, m$ stored postures:

$$ g_p = \frac{G(C_p)}{\sum_{p=1}^{m} G(C_p)}. $$

Finally, after all the stored postures are assigned weights, a single target posture, $P^*$, is found by taking the weighted sum of the stored-posture vectors:

$$ P^* = \sum_{p=1}^{m} g_p P_p. $$

With this method, the weights applied to the stored postures necessarily sum to 1, and so the target posture occupies the same range as the original stored postures, as shown in Figure 2. Also, the weights for the individual stored postures are distributed as a Gaussian of their fits to task demands. Thus, if one looks at the weights for an individual stored posture when different spatial targets are presented (rather than looking at the weights of the various stored postures when a single spatial target is presented), one finds that the individual stored postures have bell-shaped tuning curves. In this respect, the model yields response properties of stored postures that are similar to tuning curves of neurons in the brain (Knudsen, du Lac, & Esterly, 1987).  

**Feedforward correction.** A final feature of planning in Knowledge II makes it possible to bring the contact point close to the spatial target. It is not guaranteed that weighted summing of postures will achieve this goal, because mappings between postures and spatial positions are nonlinear (Bullock, Grossberg, & Guenther, 1993).

The way we cope with this problem is to (a) use forward ki-
nematics to determine where, for a given target posture, the contact point will be in relation to the spatial target; (b) then measure the signed error between the contact point and the spatial target (in extrinsic coordinates); and (c) then aim for a spatial target half as far from the spatial target as the contact point but in the opposite direction. We call this method feedforward correction. An example should clarify how the method works. Suppose the hand must be brought to a spatial target and, given the target posture that has been computed, the hand has an initial error of $\Delta x_1 = .5$ and $\Delta y_1 = .2$ with respect to the spatial target in $x$ and $y$, respectively (arbitrary distance units). A new location, biased relative to the original spatial target by $\Delta x_2 = -.25$ and $\Delta y_2 = -.1$, then serves as the spatial target. If the resulting target posture brings the hand acceptably close to the spatial target (see discussion to follow), planning stops; otherwise, it continues in the same fashion, according to the following equation, which applies in all $c$ feedforward correction cycles up to the last cycle allowed before a deadline is reached:

$$B_d(c) = B_d(c-1) + \beta(\text{TargetLocation}_d - \text{ContactPoint}_d(c)).$$

(11)

where $B$ denotes the correction bias, $d$ denotes the Cartesian dimension in which the bias is introduced (the $x$ or $y$ dimension), and $\beta$ denotes gain ($0 < \beta \leq 1$). Through simulations, we have established that the optimal value of $\beta$ is approximately 0.5; this value is optimal in the sense that it is associated with the smallest number of feedforward correction cycles needed to bring the contact point acceptably close to the spatial target.

Now what does “acceptably close” mean? The acceptable distance, $A$, between the contact point and the spatial target is assumed to be an externally imposed task constraint such as a tolerance region around a target point, as in classical studies of manual aiming (Fitts, 1954; Meyer, Smith, Kornblum, Abrams, & Wright, 1990). The units for measuring spatial error are arbitrary. They can be expressed in conventional distance units such as centimeters or, for computer simulation, as pixels. Alternatively, they can be expressed in body-scaled distance units. Studies in which human participants judge how far they can reach (Carello, Grosenky, Reichel, Solomon, & Turvey, 1989) or how high they can step (Warren, 1984) suggest that the perception of extrapersonal distance may indeed be body scaled.

An advantage of feedforward correction is that it can maximize the chance of target acquisition without overt correction based on feedback. This is advantageous insofar as it is presumably costlier to correct errors physically than computationally. In addition, the number of feedforward correction cycles provides an empirically testable index of planning time. If planning time increases with the number of feedforward correction cycles, then each cycle adds planning time. It is likely that when there is pressure to start a movement quickly, all of the necessary feedforward correction cycles are not performed in advance, so the contact point may end up farther from the spatial target than desired. In that case, overt corrections become necessary. The corrective movements must be planned in the same way as initial movements (except as noted later in the section titled Error Dead Zones and Motor Scotomas). Thus, the Knowledge model applies to the planning of corrective movements as well as the planning of initial movements.

Is there a limit to the possible effectiveness of feedforward correction? In other words, given enough feedforward correction cycles, is it guaranteed that the spatial target will be achieved? The answer is no, because there is likely to be noise in movement execution. This noise limits the accuracy with which movements can be performed, no matter how carefully the target posture may be planned. We discuss this issue later in the section titled Speed-Accuracy Trade-Offs in Aiming.

**Execution**

Once the planning process yields an acceptable target posture, a trajectory can be created for the movement from the starting posture to the target posture. Proposing a mechanism that governs the movement to the target posture is daunting because of the complexity of the musculoskeletal system and its interactions with the external environment (Enoeka, 1988; Hollerbach, 1980). Given this complexity, our proposed means of generating trajectories is likely to be oversimplified, but simplified methods are good ones against which later to compare methods that are more complex.

Our choice of a mechanism is based on one of the major descriptive properties of movement, namely, that the function relating the tangential velocity of a point along the limb-segment chain to time is generally bell shaped (see Bullock & Grossberg, 1988, for a review). Tangential velocity is the rate of change of distance between successive locations in extrinsic space. Thus, the term bell-shaped velocity profile usually refers to the rate of change of position in extrinsic spatial coordinates.

Because our model yields movements of the joints, we need to assume a characteristic velocity profile in intrinsic (joint) space. We assume that all the joints have the same bell-shaped angular velocity profile. Data consistent with this assumption have been reported by Soechting and Lacquaniti (1981) for motions of the shoulder and elbow and by Lacquaniti and Soechting (1982) for motions of the shoulder, elbow, and wrist. The eye also exhibits a bell-shaped velocity profile (Abrams, 1994; Abrams, Meyer, & Kornblum, 1989; Robinson, 1964). If the joints follow bell-shaped angular velocity profiles, it is reasonable to expect that a point along the limb-segment chain will follow a bell-shaped tangential velocity in extrinsic space. Later, we present evidence consistent with this expectation.

If one assumes that joints follow bell-shaped angular velocity profiles, one still needs to explain why they do so. The basis for bell-shaped velocity profiles has been the subject of much discussion in the motor-control literature. Some have proposed that the bell shape is optimal for minimization of costs such as mean squared jerk (Flash & Hogan, 1985) or rate of change of torque (Uno, Kawato, & Suzuki, 1989). Others, noting that features of the bell change with task contexts, have proposed that instantaneous tangential velocity reflects the momentary product of two functions of the distance remaining to the target, one increasing and the other decreasing; by manipulation of parameters of these two functions, aspects of the velocity profile,
such as its symmetry, can be varied (Bullock & Grossberg, 1988).

For us, the most natural way to motivate bell-shaped angular velocity profiles is to assume that the torques that move the joints vary sinusoidally in time: The torque begins at zero, increases to a maximum, then decreases to a minimum, and finally returns to zero, all changes being continuous. Movements of the limbs (Meyer, Smith, & Wright, 1982) and of the eyes (Abrams et al., 1989) have been modeled successfully with sinusoidal force–time profiles (see Meyer et al., 1990, for a review). The underlying force–time relations can be inferred from observed kinematics because, if a recorded movement has a bell-shaped velocity profile, the first time derivative is a sinusoidal acceleration profile, and acceleration is directly proportional to force. An important property of the sinusoidal force–time profile is that it can be globally scaled in the force domain or in the time domain to yield a bell-shaped velocity profile which is taller or shorter (force scaling) or fatter or thinner (time scaling). In the current model, we likewise assume scalability of torque–time functions.

To limit the number of free parameters in our model, we have devised a parameter-free method for generating bell-shaped angular velocity profiles. The method applies to each joint as it moves from its starting angle to its target angle. The relevant computations are shown in Figure 6. We emphasize that they are designed for ease of calculation only; no corresponding psychological processes are assumed.

In Step 1, we consider a sine function of the angle θ as it increases from (3/2)π to (3/2 + 2π)π radians. The value of sin(θ) increases from -1 to 1 and then decreases to -1. The profile within this range is bell shaped. Next, we transform these values to a function relating velocity to normalized time (Step 2); that is, we map the values of sin(θ) to a velocity profile such that the minimum of the profile is 0 rather than -1 and such that increases of θ from (3/2)π to (3/2 + 2π)π radians map to decreases of time from t = 0 to t = T:

\[ v(t) = \frac{1}{2} \left[ 1 + \sin \left( \frac{1}{3} \pi + \frac{2t}{T} \right) \right]. \tag{12} \]

In Step 3, we transform normalized velocity to cumulative distance as a function of time. Cumulative distance is an S-shaped function of time; at time t = T, it has a numerical value equal to T/2. In Step 4, we transform these cumulative distance-versus-time curves so they cover 0% to 100% of the amplitude to be covered in the time allowed. We achieve this transformation by dividing the amplitude covered as a function of time by T/2. The result is a normalized amplitude-versus-time function:

\[ \text{Norm } A_j(t) = \frac{\int_0^t v(t') dt'}{T/2}. \tag{13} \]

The resulting curve, for three values of T, is shown in Figure 6. Finally, in Step 5, we scale the values of Norm A_j(t) by the angular displacement to be produced by each joint. The angular displacement, A_j(t), of the jth joint at time t within the total time, T_j, in which joint j is supposed to move is the total angular distance, α_j, to be covered by that joint multiplied by the normalized amplitude, Norm A_j(t), for that joint:

\[ A_j(t) = \alpha_j \text{Norm } A_j(t). \tag{14} \]

Given this equation, A_j(t) equals 0 at t = 0 and increases sigmoidally to α_j at t = T. Thus, the movement covers the required angular distance in the required time and has the required bell-shaped velocity profile. This method is used for every joint.5

### Learning

Knowledge II has one final component—learning. Learning occurs through a process akin to natural selection. All stored postures enter the posture store randomly. At “birth,” all of the stored postures are randomly generated and have some arbitrary strength greater than a positive real number, Ω. Later, if a stored posture’s strength falls below Ω, it is discarded and replaced by a new, randomly chosen posture. No posture is ever deliberately selected for storage.6

The strength of a stored posture at any time t depends on the weights it has been assigned in the past. The strength, S_p(t), of stored posture p at time t is assumed to obey the equation:

\[ S_p(t) = \lambda S_p(t-1) + (1 - \lambda) g_p(t), \tag{15} \]

where g_p(t) denotes the weight of stored posture p at time t and λ is a constant (0 ≤ λ ≤ 1). According to Equation 15, a stored posture’s strength depends on how helpful it has been for move-

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5 This parameter-free method for generating angular velocity profiles cannot yield asymmetries in velocity profiles, which is a price we pay for having a parameter-free expression. Another possible price is that our movement trajectories are not maximally smooth; they do not minimize mean squared jerk, which, according to Flash and Hogan (1985), normal human actors do. When velocity profiles are maximally smooth, the ratio of peak velocity to mean velocity is 1.875. When velocity profiles are sinusoidal, the ratio of peak velocity to mean velocity is 2.000. Behavioral data are needed to distinguish between these two predictions.

6 An additional possible constraint is that the new posture cannot be too close to any existing posture. This constraint would prevent postural “black holes” that would tend to capture all the stored posture. If new postures were very similar to existing stored postures that received large weights in the past and continued to receive large weights in the future, the new replacement postures would come to have high strength. If additional new postures replaced dead postures in the same way, the only stored postures would be those that permitted reaching to the practiced area(s), and no other postures would be available for other tasks.

This learning system differs from that of Knowledge I; there planned target postures were simply accumulated. The problem with this approach was that it was unclear what to do when memory limits were met.
Figure 6. Steps used to generate movement trajectories (MT denotes movement time; see text for details).
movement planning; it is unnecessary for the stored posture to have been adopted physically to have high strength.

Making learning akin to natural selection has some advantages. Apart from the biological support such an approach enjoys (Edelman, 1987), it allows for expertise and flexibility. If particular postures consistently receive large weights, they are more likely to survive than postures that do not consistently receive large weights; thus, more postures of the often-used type come to occupy the posture store, and the speed and accuracy of movements to such postures can increase (as shown in later simulations). In addition, because new postures are introduced through a random process, the system as a whole can be pre-adapted for novel or less practiced tasks.

Evaluation of Knowledge II

We turn now to performance of the model. We focus on the ability of Knowledge II to account for known aspects of behavior. The simulations we have conducted have been rendered in part through stick-figure animations. The stick figure can bend at the hip, shoulder, and elbow and carries out reaches in the sagittal plane.

Realistic Movement Patterns

Our simulation program is interactive. A person running the program can use a computer mouse to point to a location in the work space and click on that location, indicating that the stick figure should touch that point with the hand or other contact point. The stick figure can reach for the designated spatial target, provided the target lies within the work space. Moreover, it can do so in ways that appear lifelike (Figure 7). It can reach with a hand-held tool, with an amputated limb, or with a body part other than the hand (Figure 8). People can likewise reach with hand-held implements (Flanders, Helms Tillery, & Soechting, 1992), with amputated limbs (Fraser & Wing, 1981), and with parts of the body other than the hand.

The stick figure's joints move with bell-shaped angular velocity profiles, as of course they should because they are programmed to do so. In addition, and again as a result of the way the trajectories are generated, peak velocities increase with distance covered, as commonly observed in biological motion (for a review, see Rosenbaum & Krist, in press). More surprising, the tangential velocity profile of the stick figure's hand as it moves through Cartesian space is also bell shaped or at least has proven to be in all the cases we have examined. Comparisons of the stick figure's tangential hand-path velocities with angular (joint) velocities have shown that the shapes of the two curves match essentially perfectly; the mean $r^2$ value obtained for the two curves, when tested with 100 reaches to random locations in the work space, was .9999. The latter result is important because the well-known finding that the hand follows a bell-shaped tangential velocity curve refers to the fact that the instantaneous change in the Euclidean distance covered by the hand increases and then decreases in a bell-shaped fashion. It was not obvious to us that bell-shaped angular velocity profiles for the joints would translate into bell-shaped tangential velocity profiles for the hand. The fact that they do indicates that planning in extrinsic spatial coordinates need not be inferred from the fact that the hand follows a bell-shaped tangential velocity profile (Morasso, 1981). On the other hand, if the hand follows a bell-shaped tangential velocity profile, it does not follow that the joints moved monotonically through joint space.

![Figure 7](image-url) Reaching to various spatial target locations from various starting postures. Each panel shows a reach from a starting posture to a final posture that allows the hand to be brought to within 3 pixels of the center of the spatial target (the dot). There are 11 poses in each panel, corresponding to the starting posture and 10 equally spaced intervals in the movement time, which is the same for all three joints. Equation 5 (see text) was used in arriving at the common movement time. Best-fitting values from a fit of the model to human reaching performance (see text) were used in arriving at the spatial error weight and hip, shoulder, and elbow expenses. The spatial error weight was 843, and the hip, shoulder, and elbow expenses were 215, 128, and 181, for the hip, shoulder, and elbow, respectively. The starting posture in the left-most panel is arbitrary, but the starting posture in each succeeding panel is the final posture in the panel to its left. The reason for showing successive reaches is to emphasize the fact that the stick figure can reach to any location in the work space from any other location in the work space.
with bell-shaped angular velocity profiles. This point is borne out by the observation that, in simple pointing movements, people's hands sometimes show bell-shaped tangential velocity profiles even though their joints reverse direction (Morasso, 1981).

Our model also yields relations among the kinematics of the joints that are in keeping with previous observations. Soechting and Lacquaniti (1981) showed that, during the final portions of sagittal-plane reaches, elbow and shoulder angular velocities are linearly related. They argued that such a relation suggests that reaching motions are planned in joint space (a proposition we obviously endorse) and that relations between joint motions may be explicitly specified by the motor system to reduce the number of degrees of freedom to be controlled. Our stick figure's elbow and shoulder angular velocities are linearly related whenever the elbow and shoulder accelerate or decelerate together. The reason for the correlation is that if shoulder and elbow displacements end together, they must decelerate together, and so their velocities must be related during the final portions of sagittal-plane reaches. This outcome implies that the correlation between elbow and shoulder angular velocities observed by Soechting and Lacquaniti (1981) need not be ascribed to synergistic coupling between the elbow and shoulder, though such synergistic coupling cannot be ruled out.

Compensation for Changes in Joint Mobility

One of the major achievements of the Knowledge model is its ability to satisfy task demands when normal, preferred means of moving are prevented. The phenomenon to be explained is adaptation to changes in the mobility of individual joints. Fortunately, changes in the mobility of one's joints do not leave one incapacitated. Arthritis in the hip does not prevent one from reaching for objects that would otherwise demand hip rotation, and sudden injury to the elbow does not prevent one from carrying out tasks that would otherwise require elbow rotation. These examples illustrate motor equivalence, the capacity to achieve the same goals with different effector combinations. Explaining motor equivalence is a central aim of motor-control research (Raibert, 1977; Swinnen, 1991).

The motor equivalence problem is solved in the Knowledge model without a special mechanism. The reason is that whenever the expense, $k_j$, for a joint increases (Equation 3), the relative contributions of other joints increase. Representative compensation patterns are shown in Figure 9. These simulations were generated by increasing the expense for the hip or for the elbow. When the hip's expense increased, candidate postures that demanded large displacements of the hip had high travel costs, so those candidate postures received small weight, and the target postures that were chosen were ones that could be reached with small hip excursions. Similarly, when the elbow's expense increased, candidate postures that demanded large displacements of the elbow had high travel costs, so those candidate postures received small weight, and the target postures that were chosen were ones that could be reached with small elbow excursions. In both cases, the compensation occurred without complex feedback correction. As far as we know, this method of compensating for changes in joint mobility is simpler than any used before (Mussa-Ivaldi, Morasso, & Zaccaria, 1988; E. Saltzman & Kelso, 1987).

Speed Effects

Movement patterns differ when movements to the same spatial location are carried out at different speeds. One study that

\footnote{The capacity of our system to achieve instant compensation for changes in the mobility of particular joints contradicts a statement by Bullock, Grossberg, and Guenther (1993) that systems that rely on optimization of spatial trajectory formation given target-posture mappings cannot compensate for the "freezing" of joints without extended relearning of system parameters.}
Figure 9. Compensation for increased expense of elbow rotation (middle panel) or hip rotation (bottom panel) in comparison with reaching with normal joint expenses (top panel). The starting posture and spatial target location (in front of the knee) are the same in all three panels. Model parameters and other conventions are the same as in Figure 7.

demonstrated this feature of motor performance was reported by Rosenbaum et al. (1991). Their participants oscillated the tip of the right index finger in the horizontal plane and could wave the finger, hand, or forearm in any preferred combination. As the speed of oscillation increased, the relative contribution of the finger increased and the relative contribution of the forearm decreased; however, as the speed of oscillation decreased, the relative contribution of the finger decreased and the relative contribution of the forearm increased. These results were repli-

cated by Vaughan, Rosenbaum, Moore, and Diedrich (1993) in a finger tapping task and by Meulenaer, Rosenbaum, Thomassen, and Schomaker (1993) in a drawing task. Similar results were also obtained by Brüwer and Dean (1993), who found that, in a horizontal planar movement task that could be achieved with different combinations of shoulder, elbow, and wrist movements, the shoulder contributed less and the wrist contributed more as speed increased. Such results are predicted by the Knowledge model because, when required movement time is reduced, disproportionately high travel costs are assigned to stored postures that demand large rotations of joints with high expense, and so postures with lower travel costs are favored. Relevant simulations for sagittal-plane movements are shown in Figure 10.

Changes in Movement Curvature and Joint Contributions Depending on Movement Direction

As stated earlier, we assume that the motion of the current posture vector is a straight line through joint space, which is consistent with the view that the trajectory of the current posture vector obeys the principle of least action in intrinsic spatial coordinates. An important consequence of this assumption is that a point on the limb-segment chain need not follow a straight line as it moves through extrinsic space, which implies that the point on the limb-segment chain need not follow the principle of least action in extrinsic spatial coordinates. The Knowledge model predicts, in fact, that the spatial path followed by a point on the limb-segment chain should vary in curvature depending on the direction of motion relative to the body.

Consider a reach that demands a vertical or horizontal displacement of the hand in the sagittal plane. As seen in Figure
11, the curvature of the hand path is greater for the vertical movement than for the horizontal movement. The least costly way to move the hand between the two vertical locations is to rely primarily on the shoulder, secondarily on the elbow, and hardly at all on the hip, so the hand path is highly curved; if the hip were more involved, the hand could move in a straight path, provided there were counterrotations of the shoulder and elbow. By contrast, the least costly way to move the hand between the two horizontal locations is to rely about equally on the shoulder and elbow and, again, hardly at all on the hip; with this combination of effectors, the hand path is straighter. Such a difference in hand-path curvature was reported by Atkeson and Hollerbach (1985), who recorded reaching movements of human participants. These authors also found that there was more shoulder movement in the case of horizontal hand displacement than in the case of vertical hand displacement. This outcome is also predicted by the model, as shown in Figure 11.

Starting Position Effects

The Knowledge model predicts that different postures should be adopted when the hand (or other contact point) is brought to the same spatial target location from different starting postures. This outcome is predicted because target postures are chosen with respect to travel costs, and travel costs are defined with respect to starting postures. Starting position effects have been reported, both when participants knowingly adopt different starting postures (Cruse, Brüwer, & Dean, 1993; Fischer, Rosenbaum, Loukopoulos, & Szymkowiak, 1994; Gordon, Ghilardi, & Ghez, 1992; Wright & States, 1992) and when participants' hands are surreptitiously moved to new starting positions by the experimenter (Jaric, Corcos, & Latash, 1992; Sittig, Denier van der Gon, & Gielen, 1987).

A more detailed prediction of the Knowledge model is that the effect of the starting posture on the posture adopted at a spatial target should be such that the target posture minimizes the travel cost from the starting posture. The studies just cited do not permit a direct test of this prediction, so a study was conducted in our laboratory to do so (Fischer et al., 1994). Participants reached for a fixed spatial location from several starting postures. The main dependent variable was the correlation between the angle of any given joint when the participant adopted a target posture to reach the fixed spatial target and the angle of that same joint when the participant was in the starting posture. The model predicted that the correlation would be highest for the joint whose rotation was most costly, because for that joint it would be most crucial to keep the final angle of the joint close to its starting angle. The hip was found to have higher correlations than the shoulder or elbow, which was consistent with the prediction because the hip's expense was independently estimated to be higher than the shoulder's and elbow's (see Figure 7).

Setting In

A corollary of the starting position prediction is that, if a reaching task is performed repeatedly, the variability of postures adopted at the spatial targets should decrease. Such "set-
tling in" is predicted because reaches made to a series of target locations should initially be affected by the posture that happens to be adopted before the task begins, but as the task continues (as the target locations are reached for repeatedly), the influence of the arbitrary starting posture should fade.

This prediction was also tested by Fischer et al. (1994). Participants reached for a series of spatial targets, reaching for the same targets 10 times without stopping. As predicted, the target postures had greater variability in the first cycle than in the later cycles.

Adverse Effects of Reduced Information About Starting Positions

According to the Knowledge model, limited information about starting positions should result in target postures with higher or more variable travel costs than normal, and lower spatial accuracy than normal. Higher or more variable travel costs are predicted because travel costs are assumed to be computed with respect to starting postures. Lower spatial accuracy is predicted because movement trajectories are programmed, taking into account the amplitude and direction to be covered from the starting posture; if the starting posture is uncertain, then amplitude, direction, or both may be registered incorrectly, and the generated movement may be inaccurate as well.

Many results are consistent with these predictions. Aimed hand movements are more accurate when the hand can be seen before moving than when the hand cannot be seen before moving (Prablanc, Echallier, Jannenrod, & Komilis, 1979). Also, catching is more accurate when the participant has sight of the catching hand than when the participant does not have sight of the catching hand (Smyth & Marriott, 1982), although this effect is larger for skilled catchers than for less skilled catchers (Savelsbergh & Whiting, 1988). The reduction or alteration of proprioceptive input also impairs positioning accuracy (Keele, 1968), consistent with the expectation that greater uncertainty about limb position impairs planning of target postures.

That alteration of proprioceptive information about starting posture per se impairs aiming was shown by Sittig et al. (1987). These investigators applied vibration to the tendons of the forearm muscles, knowing that such vibration induces misperception of limb position (McCloskey, 1973). When participants tried to point to a seen target (without visual feedback), they exhibited consistent aiming errors. This outcome fits with the assumption that starting positions are taken into account during planning.

Another study that confirms the importance of starting position information was reported by Ghez, Gordon, Ghilardi, Christakos, and Cooper (1990). They studied blind positioning performance in patients with severe large-fiber sensory neuropathies affecting both arms. The arm movements of these patients were often misdirected as soon as the hand departed from the home position.

Additional evidence for the role of proprioception in registering starting postures was presented by Hasan and Stuart (1988). They cited several sources of support for the notion that limb movements (or limb final positions) are programmed with respect to starting postures. One of the best known of these studies was Polit and Bizzi's (1979) classic demonstration that if a monkey's deafferented arm is displaced before a blind movement is made to a target, systematic errors ensue in the final position achieved. This result, as well as the others just reviewed, supports the hypothesis that movements are planned with respect to starting positions, as assumed in the Knowledge model. Any model that assumes that target positions are programmed without regard to starting positions predicts that reduced information about starting positions should have a negligible effect on positioning accuracy. Early versions of the mass-spring model, which proposed setting of target muscle stiffnesses without regard to starting configurations, could be rejected on this basis (Politt & Bizzi, 1979).

Growth of Variability During Series of Blind Positioning Movements

In the experiment of Fischer et al. (1994) mentioned previously, participants could see the spatial targets to which they moved, and they had visual feedback about their starting and ending (target) postures. Suppose instead that participants make aimed hand movements to series of target locations without visual feedback, and suppose that neither the spatial targets nor the hand can be seen during the series of aiming movements. As reported by Bock and Eckmiller (1986), the variability of positions adopted in this situation increases as the task continues. This result seems to contradict the settling in prediction because the essence of settling in is shrinking variability. Can the Knowledge model account for both results?

In the preceding section, we noted that removal of visual feedback increases uncertainty about starting postures. This observation helps explain Bock and Eckmiller's (1986) result from the perspective of our theory. In a series of aimed hand movements, each target position is also the starting position for the next movement. Thus, removing visual feedback adds uncertainty about starting postures as well as target postures. Given that uncertainty about starting positions increases target-position variability, as discussed in the last section, where the hand ends up in successive blind reaches should become increasingly variable as the task continues.

Bock and Eckmiller (1986) argued that their increasing-variability result implies that movements are programmed in terms of direction and distance but not location. In the Knowledge model, movements are programmed initially in terms of target postures, and then directions and distances of motions come into play when trajectories are created. The Knowledge model therefore allows for programming of directions and distances in the trajectory-specification stage, although it does not allow for programming with respect to direction and distance in the target-posture specification stage. From the standpoint of the model, therefore, Bock and Eckmiller's result does not require denial of planning with respect to locations.

Anticipatory Effects in Reaching

According to the Knowledge model, the posture to be adopted at the end of a reach is known before the reach begins. Recent evidence indicates that final postures are known before
the hand leaves the start location (Rosenbaum, Vaughan, Barnes, & Jorgensen, 1992). In Rosenbaum et al.’s study, participants reached out from a fixed starting position to grab a bar and move it as quickly as possible to a target position; the bar was oriented vertically or horizontally. The trial began with a signal indicating where the bar should go; this signal also served as the “go” stimulus. Participants were instructed to minimize the time between appearance of the signal and transport of the bar to the target location; they were not told to minimize the time to leave the starting position. Nonetheless, different departure times were observed when the bar was spontaneously grabbed in different ways. For example, when the bar was oriented vertically and was supposed to be brought to one target position, the mean departure time for the hand was different when participants spontaneously grabbed the bar with an underhand grip than when they spontaneously grabbed the bar with an overhand grip. Thus, the posture that was adopted when the bar was grabbed one way or the other was established before the hand left the starting position. This result supports the prediction of the Knowledge model that target postures are known before movements begin.  

Preshaping of the hand during reach-and-grasp movements provides another source of support for the view that actors have advance knowledge of forthcoming postures (for a review, see Jeaanerod, in press). The distance between the thumb and index finger varies according to the width of the object to be grasped, and the orientation of the hand varies according to the object’s orientation. Because such modulations occur well in advance of contact with the object, the final configuration of the hand and arm is represented before the movements are completed.

Strokes in Writing and Drawing

Studies of handwriting and drawing have revealed that graphic production is achieved with series of strokes. Strokes have been inferred from errors in graphic production (Margolin, 1984; Margolin & Wing, 1983) and from the timing of pen-tip trajectories (Lacquaniti, Terzuolo, & Viviani, 1983, 1984; Morasso & Mussa-Ivaldi, 1982; Plamondon, 1993; Teulings, in press; Wright, 1993). Of special importance here, strokes have been found even when the pen tip moves in an apparently continuous fashion (for reviews, see the references just cited and Rosenbaum, 1991, chap. 7).

Although strokes have long been recognized to play a central role in graphic production and to support hierarchical models of graphic production (van Galen & Wing, 1984), the reason for their existence has been unclear. The Knowledge model explains them. According to the model, a movement to a target posture can involve only monotonic trajectories through joint space. Whenever there is a nonmonotonic joint path, the set of joint angles at the inflection point must have been specified ahead of time as a distinct target posture. Distinct movements (strokes) are therefore needed between successive postures whenever there is a change in movement direction requiring a joint reversal.

Let us amplify this argument. Suppose a person wishes to draw a closed curve (a continuous curve from the starting location of the pen tip to another location and then back to the starting location). Because the starting location of the pen tip also serves as the pen tip’s ultimate target location, the starting posture can be the ultimate target posture. However, if the starting posture and target posture are the same, no movement is possible, simply because movement must occur from one posture to another posture. An intermediate target posture is therefore necessary. Whatever the intermediate target posture, it must be one in which one or more joints reverse direction, because the posture from which the intermediate posture is approached is the same as that to which the next movement is directed. Hence, two strokes are required in drawing the closed curve, one from the starting posture to the intermediate target posture and one from the via target posture to the final target posture. (In generating a particular desired curve, it is necessary to specify the location to which the pen tip must be transported when the intermediate target posture is adopted [as we discuss later in the Simulating Handwriting section].) The important point for now is that the curve cannot be drawn without distinct strokes, even if the trajectory as a whole seems, on cursory inspection, to be produced as one continuous movement. The model predicts, in fact, that the number of strokes should be correlated with the number of joint reversals and that strokes should end when joints reverse direction. As far as we know, no published data are available yet to permit an evaluation of this prediction.

Undershooting and the Tendency to Make Larger Extent Errors Than Direction Errors

Participants typically make larger errors in the extents (amplitudes) of blind positioning movements than in the directions of such movements, and they tend to undershoot more than overshoot in the extents they produce (Ghez et al., 1990; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979; Sidaway, Sekiya, & Fairweather, 1993; Sidall, Holding, & Draper, 1957). This result is predicted by the model; the prediction is based on considerations of spatial error costs and travel costs for candidate stored postures.

Consider Figure 12, which shows the distribution of costs that would be assigned to each of 12 stored postures, assuming a movement from the start location and the requirement to bring the end effector to the target location (the black dot at the center of the array). The contact point for each stored posture is at the center of one of the numbered circles, and the number in each circle is the total cost of the stored posture. The total cost associated with each stored posture is the weighted sum of the travel cost, treated as the Euclidean distance between the start location and the location occupied by the circle’s center, and the spatial error cost, treated as the Euclidean distance between the

8 An important feature of this result was that the reaction times differed even when the bar was in the same position, the target location was constant, and the “go” signal was the same. Previous studies which have claimed that reaction-time differences for different reaches reflect changes in motor preparation have been challenged on the grounds that motoric and perceptual aspects of the tasks were confounded (Goodman & Kelso, 1980). There was no such confound in the study of Rosenbaum, Vaughan, Barnes, and Jorgensen (1992).
the tendency to make extent errors rather than direction errors. The reason is that as the weight assigned to travel costs approaches zero, the only determinant of the total cost of a stored posture is how far its contact point is from the spatial target. Therefore, for a situation like the one shown in Figure 12, the set of contact points having equal total costs should occupy a circle, rather than an ellipse, around the spatial target. We know of no published data that bear on this prediction directly, so its evaluation must await future tests.

Another prediction, which follows directly from what we have just said, is a bit more surprising. According to the Knowledge model, large effectors should be relied on more as tasks require more spatial accuracy. This prediction follows from the fact that when spatial accuracy needs to be maximized, making it less important to minimize travel costs, those effectors that have high travel costs (large effectors) should be used more. Again, we know of no previous studies that bear on this prediction directly, so the prediction will have to be judged through new behavioral experiments. There are data that indirectly support the prediction, however. Consider an experiment in which participants bring the tip of the index finger to a small or large target; the center of the target can be reached entirely through rotation of the index finger, entirely through rotation of the wrist, or entirely through rotation of the elbow. Assuming that the travel cost of the index finger is less than the travel cost of the forearm for equivalent distances and times, the model predicts that the contribution of the index finger should diminish as the required spatial accuracy increases, whereas the contribution of the forearm should increase. Such an experiment was conducted by Rosenbaum et al. (1991). Participants in this experiment, which was mentioned earlier, had to move the tip of the index finger back and forth so that the finger crossed two targets a variable distance apart at a variable rate. Each task could be performed entirely through rotation of the index finger, entirely through rotation of the wrist, or entirely through rotation of the elbow. The main result was that participants used more forearm rotation as frequency decreased and as distance increased. The outcome indirectly supports the prediction that larger effectors should contribute more when greater spatial accuracy is demanded, because slow movements (movements made at low frequency) are associated with more spatially accurate displacements, whereas fast movements (movements made at high frequency) are associated with less spatially accurate displacements. Further work is needed to show whether the main source of this effect was speed or spatial accuracy.

A final comment about undershooting and the tendency to make larger extent errors than direction errors is that we are not claiming that the Knowledge model is the only model that can predict these effects. Any model that places a premium on reducing effort might also be able to predict these results, although we are unaware of any model that has done so.

**Error Dead Zones and Motor Scotomas**

The Knowledge model makes a prediction that at first glance, is unsettling. Because of the nonlinearity of posture-location mappings and the fact that there are only a finite number of stored postures, it is theoretically possible to have locations...
within the work space to which a contact point cannot be brought. Before our introduction of feed-forward correction in Knowledge II, we were frustrated in watching our stick figure simulations because there were some locations to which the stick figure could not bring the hand; it would often miss the location by a noticeable (although always small) amount. The introduction, in Knowledge II, of feedforward correction ameliorates this problem considerably, but it still remains: There are still places in the work space to which the stick figure cannot bring its hand, although, again, none of these places is far from a place that can be reached (just a few pixels at most). Where these unreachable points are is difficult to predict.

One reaction to this phenomenon might be to discount its practical importance. How often does one have to reach for an exact point, one might ask? Perhaps in real life, when reaches are made to three-dimensional objects with three-dimensional effectors, the problem is nothing more than a theoretical curiosity. It is interesting to observe, however, that an "error dead zone" was recently reported for aiming performance (Wolpert, Miall, Winter, & Stein, 1992). In Wolpert et al.'s study, subjects (both human and monkey) performed a compensatory tracking task. They moved a manipulandum to bring a cursor in line with a target whose position was shifted randomly. The experimenters introduced an artificial dead zone into the system by adding a distance threshold below which position error was not displayed. The question was how large this artificial dead zone had to be before a difference could be detected between errors made when the dead zone was present and when the dead zone was absent. If the difference emerged when the artificial dead zone was small, this would indicate that subjects were sensitive to error feedback, however, if the difference emerged only when the dead zone was large, this would indicate that subjects were relatively insensitive to the error feedback. In fact, the dead zone had to be quite large before a difference could be found (about 7 screen pixels, or .265° of visual angle). Thus, there was some distance over which subjects were unable or disinclined to compensate for disparities between the positions of the cursor and target. The error dead zone was not due to simple perceptual limits, motor limits, or artifacts of the procedure or measurement (see Wolpert et al., 1992, for details). The authors concluded that some mixture of perceptual and motor factors was responsible for the dead zone. The Knowledge model indicates what this set of factors might be.

Another study that provides support for this counterintuitive prediction of the Knowledge model was conducted by Ghez et al. (1990). They discovered what they called motor scotomas, small areas in extrinsic space to which the hand was rarely (or never) brought during blind positioning movements (even though those areas were tested no less often than others). Ghez et al. remarked on the surprising nature of these gaps and admitted that they could not explain them. The Knowledge model may do so.

We wish to point out that if the Knowledge model does explain motor scotomas and error dead zones, it does not require such gaps to occur when participants have full feedback and the opportunity to correct errors as precisely as they wish. Clearly, a normal individual can bring his or her fingertip to any point in the workspace. What the Knowledge model can explain is that small, systematic errors are likely to occur when movements are made ballistically. The model allows for the fact that errors can be corrected when the opportunity for error correction exists, because when there are very small errors and when motion of several degrees of freedom is needed to correct them but they cannot be reduced through normal feedforward correction, the errors can be reduced by trial and error. When very small errors exist and the motion of one degree of freedom can correct them, the errors can be reduced by trial and error or by deliberate homing in.

**Speed–Accuracy Trade-Offs in Aiming**

The last two sections have concerned errors in aiming performance. Earlier, when we discussed feedforward correction, we said that the model allows for a reduction in the number of corrective movements that are needed as more feedforward corrections are made. We also noted that there should be an upper limit on the effectiveness of such feedforward corrections: Even if the feedforward correction process yields a target posture whose contact point is expected to arrive at a location that is acceptably close to the spatial target, the contact point may not reach that location when the movement is carried out because of noise in execution. We have not yet specified the nature of the noise, however.

Recall that the execution component of our model is based on the assumption that movements of the joints are driven by torques that vary sinusoidally in time. This torque profile is directly analogous to the force profiles that have been assumed by other authors (Meyer et al., 1982, 1990). The assumptions we make about noise in the execution component of the Knowledge model are directly analogous as well. Meyer et al. assumed that noise increases with movement velocity. They also assumed that the standard deviation of the hand's end position is proportional to the distance covered by the hand and inversely proportional to the duration of the hand's movement. They showed that Fitts' Law can be viewed as the solution to the problem of finding an optimal time and distance given this underlying variability when the task is to reach a target as quickly as possible (Meyer, Abrams, Kornblum, Wright, & Smith, 1988). Furthermore, they showed that detailed predictions concerning speed–accuracy trade-offs can be confirmed on the basis of this premise. Given the success of their approach, and given that our assumptions about execution follow theirs, we can similarly account for speed–accuracy trade-offs in our model.

Two more points deserve mention. First, because we are dealing with angular motions of the joints rather than rectilinear motions of the hand, it remains to be seen whether the speed–accuracy trade-offs commonly observed for the hand in extrinsic space can be generated with noisy motions of individual joints. Additional simulations are needed to determine whether they can.

Second, the model of Meyer et al. (1988) does not cover the planning process for forthcoming reaches; rather, it represents speed–accuracy trade-offs in terms of completed solutions to an optimization problem. Because the planning process is explicit in the Knowledge model, the model predicts errors due to planning as well as execution. We have already outlined the kinds of errors that occur from planning. These stem from the nonlinear
mapping between postures and extrinsic spatial locations (error dead zones and motor scootomas) and from the inclusion of travel costs as well as spatial error weights in the derivation of target postures (undershooting and the commitment of larger extent errors than direction errors). Errors associated with execution limit the effectiveness of planning. When enough feedforward correction cycles have occurred to ensure that, in planning, the contact point will come to a location that is acceptably close to the target location, the contact point may, in fact, not reach the target location if movement noise is too large. Thus, after a certain number of feedforward correction cycles, although the planned end location of the contact point may move closer and closer to the spatial target, there will be no observed improvement in achieved accuracy. Moreover, the number of feedforward correction cycles needed for asymptotic accuracy should decrease as movement noise increases. Because we assume that movement noise increases with velocity, the prediction is that if a planned movement must cover a large distance in a small amount of time (a high velocity movement), there should be little or no benefit of additional feedforward correction cycles. By contrast, if a planned movement can cover a small distance in a large amount of time (a low velocity movement) or even in a small amount of time (a somewhat higher velocity movement), there should be a benefit of additional feedforward correction cycles.

Qualitative support for this pair of predictions comes from studies concerning the relation between initiation time and movement time in manual aiming performance. Fitts and Peterson (1964) and Ellis (1973), studying fast aiming movements covering large distances, found no relation between the time to initiate the movements and the accuracy of target attainment. However, Klapp (1973), studying fast aiming movements covering short distances, found a significant positive relation between movement initiation time and accuracy. This pair of outcomes accords with the hypothesis that feedforward correction cycles can ensure more accurate movements, but only to the extent permitted by movement noise.

Some might argue against the interpretation we have just given on the grounds that the absence of a correlation between initiation time and movement time merely reflects a greater opportunity for correction in rapid long-distance movements. Perhaps participants plan less for rapid long-distance movements because they expect to make corrections (Klapp, 1973). Our reaction is that the presence of corrections supports the assumption that rapid, long-distance movements are variable. Furthermore, one must ask why participants would choose to make overt corrections rather than plan carefully. If making overt corrections were less costly than planning, it would make sense to move and correct rather than to plan in more detail. We doubt that this reflects the true state of affairs, however. We believe that it is less costly to plan carefully and make few overt corrections than to plan roughly and make many overt corrections. The only reason to limit planning is if movements are noisy.

Growth of Expertise

The Knowledge model predicts that after a random distribution of postures "at birth" (Figure 13), the distribution can change depending on experience (Figure 14), such that the mean spatial error of target postures decreases with practice (Figure 15) and mean planning time decreases with practice (Figure 16). Given these theoretical possibilities, the model can account for the superiority of movement reproduction toward regions of extrinsic space to which reaches are often directed (e.g., directly in front of the body) in comparison with regions of extrinsic space to which reaches are not often directed (e.g., behind the body or on the opposite side of the body). Results consistent with these predictions have been reported by Fitts (1947), Steinmach and Larish (1980), and Fisk and Goodale (1986). Likewise, the model can account for the reduction in initiation times for highly practiced reaches in comparison with less highly practiced reaches (Newell & Rosenbloom, 1981; Requin, 1977; Rosenbaum et al., 1992). Furthermore, the model can account for the fact that less energy-demanding modes of performance are adopted as skills develop (Sparrow & Zirzarry-Lopez, 1987). This outcome follows from the fact, confirmed in additional simulations, that travel costs for reaches to repeatedly tested spatial regions decline as more reaches are made to those regions.

There are aspects of the growth of expertise for which the model does not account, however. First, it does not account for the learning of timing. That is, it does not explain how pianists improve the timing of their keystrokes or how golfers improve the timing of their swings. Such learning would have to occur in a module that provides input to the posture-evaluation system. We implied the existence of such a module when we discussed time in the calculation of travel costs. In that discussion, we implied that a system outside the posture-evaluation system would have to specify whether times for joint motions should be defined externally or internally and whether the joints should
begin and end their motions together. How these decisions are reached and how times are learned are topics to be taken up in the future.

The model also fails to account for learning of new perceptual-motor mappings such as those that occur in lens or prism adaptation. Other models can explain such learning (e.g., Bullock et al., 1993) and so can be judged superior to the Knowledge model from this point of view. However, it is possible that the Knowledge model can explain perceptual-motor learning if one assumes that new mappings are formed at a different level from the one in which postures are stored and evaluated. The reasoning is as follows. It would be undesirable to have to change the mappings of postures to locations in extrinsic space, because the mappings are nonlinear. On the other hand, it would be desirable for new mappings to be formed between low-level visual representations of extrinsic locations and higher level representations of extrinsic locations, because those new mappings could be formed through linear transformations of already-existing mappings (assuming conventional lenses or prisms). After the new mappings have been learned, registered extrinsic locations could provide useful inputs to the posture-evaluation system; however, before the new mappings are formed, the registered extrinsic locations fed to the posture evaluation system would be improperly related to the true locations to which movements must be directed. This proposed method of adaptation must still be evaluated in detail.

Awkwardness During Growth spurts

The physical awkwardness that reportedly occurs during growth spurts can also be understood with the model. When limb lengths change, joint-angle combinations have different effects than they did before. The location of the hand or other contact point reached with a given set of joint angles changes as the lengths of the limb segments change (see Equation 6). Hence, postures that were previously stored because of their utility for reaching frequently tested spatial locations are no longer appropriate, and so those joint-angle combinations (i.e., those posture representations) diminish in strength, making room for new, better adapted combinations. Until a new set of posture representations has been stored, the speed and accuracy of reaching should be worse than before.

Two additional remarks should be made about the claim that the Knowledge model can explain awkwardness during growth spurts. First, we know of no formal studies demonstrating that such physical awkwardness exists, so we are relying on anecdotal evidence when we say that it does; formal experiments can, of course, be conducted to test the belief that people are physically clumsy when they grow quickly. Second, we are not claiming that the Knowledge model is the only model that can predict changes in coordination when body size changes dramatically. Although we know of no theory that has explicitly predicted awkwardness during growth spurts, any model for which learned commands or mappings must change as the size of the body changes would probably be able to predict this effect.

Individual Differences

Individuals vary in their perceptual-motor abilities. Variation in parameters of the Knowledge model can predict such differences. Consider the number of possible stored postures. As shown in Figure 17, simulated individuals with large posture stores can carry out positioning movements more quickly and accurately than simulated individuals with small posture stores. Although this result is specific to aiming, it illustrates how variation of a parameter within the Knowledge model can affect performance quality. Clearly, variation of other parameters can likewise lead to changes of performance that might be related to individual differences (e.g., variation of \( \lambda \), the learning rate parameter, in Equation 15).

Fitting the Model to Data

Our attempts to demonstrate the power of the Knowledge model have so far been largely qualitative. We have tried to develop a model that achieves what normal, skilled actors can achieve, through means that are physically, physiologically, and psychologically plausible. The ultimate test of the model, however, is not simply whether it can capture qualitative features of performance but whether it can account quantitatively for human (and animal) behavior, especially in tasks for which the model makes distinct predictions. Toward this end, we have conducted experiments in which people carry out reaching tasks, their movements are recorded, and their kinematic data are fitted with the model. Although this work was in progress at the time this article was written, the qualitative outcomes, some of which have been described here (Fischer et al., 1994), were
Figure 15. Mean spatial error as a function of experience. The unit of spatial error is the mean Euclidean distance between the hand and the spatial target divided by the largest possible Euclidean distance between any two spatial targets in the work space. In these simulations, spatial targets were presented at random locations within Area A (top panel), C (middle panel), or E (bottom panel) to a stick figure that had reached to 600 random targets within Area A, C, or E. As a means of obtaining repeated observations without altering what had been learned before the test phase, learning was prevented during testing by setting $\lambda$ to 0. After 100 targets had been presented within each test area, learning was reactivated. Learning was stopped for the stick figure after 200 learning trials, after 400 learning trials, and after 600 learning trials (horizontal axis). Spatial error was smallest within the target area that had been experienced most. Furthermore, spatial error associated with targets in the learned areas continued to decline with added experience at a cost for targets in the unfamiliar areas. Areas C and E suffered as experience was gained in A; Areas A and E suffered as experience was gained in C; and Areas A and C suffered as experience was gained in E. The starting posture was held constant in all reaches; it is the one shown in Figure 13. Areas A, C, and E are shown in Figure 14.

...encouraging. When the model was fitted to data from a study in which seated participants wore markers on the knee, hip, shoulder, elbow, and wrist and reached to each of 12 targets in a $3 \times 4$ matrix of locations within the sagittal plane, it was able to account for more than 96% of the variance in the target postures that participants adopted (Vaughan, Rosenbaum, Loukopoulos, & Engelbrecht, 1994).9

9 The fit was achieved by supplying to the model the Cartesian locations of the joints in the starting posture, along with the final Cartesian location of the wrist in that trial. The wrist location was then treated as a spatial location to which a reach had to be directed. For purposes of fitting the model, the number of stored postures was fixed at 666. Learning was turned off; that is, $\lambda$ was set to 0 so that the model performed only at a steady state of experience. In addition, the spatial error constraint was set at 5 pixels and the maximum number of feedforward correction cycles was set at 20. The value of 5 pixels was chosen because it corresponded to the uncertainty in final wrist location resulting from measurement error in the experiment; we reasoned that the stick figure should not have to bring its wrist any closer to the spatial target than measurement error justified. The value of 20 feedforward correction cycles was expected, and indeed proved, to be large enough never to interrupt the feedforward correction process before the spatial error constraint was achieved. The fitted model had four estimable parameters: the three joint expenses, $k_j$, and the spatial error weight ($w_i$ in Equation 5). The spatial error weight was bounded between 0 and 1, and the joint expenses were bounded between 0 and 222, based on the assumption that the largest possible optimal time, $T_j^*$$(\alpha_j)$, for any joint $j$ to traverse an angle of $\alpha_j = 90^\circ$ is 1,000 ms, in which case it follows from Equation 3 that the largest possible value of $k_j$ is 222. The result of the simulation was that best-fitting joint-expense values were 215, 128, and 181, for the hip, shoulder, and elbow, respectively, and the best-fitting spatial error weight ($w_i$ in Equation 5) was .843. The proportion of variance from the average joint angle accounted for with this set of parameters was $r^2 = .9635$. As a means of evaluating the stability of these parameter estimates, the model was tested with 12 different random sets of 666 stored postures. (The values just reported are the means over these 12 runs.) The 95% confidence intervals for the best-fitting parameter estimates over the 12 runs were ±4.307, ±4.362, and ±6.037 for the hip, shoulder, and elbow expense, respectively, and ±0.014 for...
Figure 16. Number of feedforward correction cycles needed to achieve an acceptably small spatial error (3 pixels) as a function of experience. These data were obtained in the same series of simulations as the ones yielding the results shown in Figure 15. As seen here, the number of feedforward cycles needed for tolerable performance was smaller for targets in familiar areas than for targets in unfamiliar areas. The number of feedforward cycles declined with practice, and, after 600 learning trials, the number of feedforward cycles for unfamiliar areas increased the more remote they were from the familiar area. This outcome can account for the fact that planning time decreases as tasks become more practiced.

Extensions and Further Issues

This section deals with extensions of the model and additional issues we plan to address.

Trajectories as Schedules

In the preceding discussion, whenever we mentioned the model's method for trajectory generation we were careful not to say that, on generating a trajectory, the actor necessarily carries it out. Because the trajectory is established before a movement occurs, there is no need to begin moving as soon as the trajectory is assembled. Instead, the assembled trajectory can be used to check that the movement will occur as desired. One reason to check the trajectory is to determine whether the movement will result in a collision with an obstacle. Pursuing this idea, we have recently developed a method for avoiding obstacles based on the Knowledge model (Loukopoulos, 1994; Loukopoulos, Rosenbaum, Meulenbroek, & Vaughan, 1993).

The assembled trajectory can also provide a schedule of limb positions (Rosenbaum, 1985). Such a hypothesized schedule can explain one of the most intriguing results of motor-control research. Bizzi, Accornero, Chappelle, and Hogan (1984) trained monkeys to make forearm movements in the horizontal plane toward a spatial target. The monkeys could not see their arms because a barrier was placed over the arms' movement path, nor could they feel their arms because the dorsal roots of their spinal cords were cut. As the monkeys moved toward the target, their forearms were sometimes displaced by an external device in the direction of the target. One might think that, under this circumstance, the arm would simply cover the remaining distance to the target, getting there sooner than normal. In fact, it did the opposite. It returned to the position it would have occupied at the time of the disturbance and then proceeded to the spatial error weight. These confidence intervals can be expressed as proportions of the possible range of each type of parameter, that is, as proportions of the range of possible joint expense (0 to 222) and as a proportion of the range of possible spatial error weights (0 to 1). The confidence intervals translate to ±0.0194, ±0.0196, and ±0.027, for the hip, shoulder, and elbow expense, respectively, and ±0.014 for the spatial error weight. The smallness of these values indicates that the model fits were insensitive to random variation in stored postures. Further fits of the models to other numbers of stored postures yielded similar profiles of best-fitting parameters (Vaughan, Rosenbaum, Loukopoulos, & Engelbrecht, 1994).
limb follows a trajectory defining a series of virtual equilibrium positions. In terms of scheduling, a "list" was generated indicating where the limb should be and when it should be there. If the limb happened to be displaced, it was simply returned to where it should have been according to this schedule.\textsuperscript{10}

It is interesting to note that Haggard and Wing (1991) recently presented a finding that can be interpreted in the same way (see Figure 18). In their study, human participants reached forward to grab an object, beginning the reaches with the tips of the thumb and index finger close together. During the movements, the distance between the thumb and index finger increased and then decreased as the fingers closed around the object, as usually occurs in reach-and-grasp movements (Jeannerod, in press). On some trials, an external device was used to pull the arm away from the target. Haggard and Wing observed that, as the arm was pulled back, the distance between the thumb and index finger decreased, approximating the distance typically observed between the thumb and index finger when the arm was in this more retracted position. Once the forward movement resumed, the distance between the fingers increased again and finally decreased during the grasp of the object. A similar observation was made by Paulignon, MacKenzie, Marteniuk, and Jeannerod (1990), who noticed that the separation between the fingers changed as participants repositioned their arms while trying to reach for an object whose position suddenly shifted.

The coupling of hand position and finger–thumb distance observed by Haggard and Wing (1991) and by Paulignon et al. (1990) can be understood in the Knowledge model in terms of scheduled postures. When the joints of the arm are in one set of angles, the joints of the fingers need to be in a corresponding set of angles to complete the posture. Coupling of arm and finger positions may be observed, then, because arm and finger positions constitute single, scheduled postures.

\textbf{Coarticulation Effects}

Knowledge II so far includes planning only for immediately forthcoming movements; it allows for planning of movement \(n + 1\) after completion of movement \(n\) but does not allow for planning of movement \(n + 2\) or beyond after completion of the \(n\)th movement. In principle, there is no reason why such long-range planning must be omitted from the model; it has been left out only for simplicity. An important reason to consider planning of more than immediately forthcoming movements is that performance is marked by widespread anticipatory (coarticulation) effects. The way a syllable is pronounced depends on what syllable will be said later (Kent, 1983), the way typewriter keys are struck depends on what keys will be struck next (Rumelhart & Norman, 1982), and the way strokes are made in handwriting depends on what strokes will be subsequently produced (Teulings, in press; for reviews, see Rosenbaum,\textsuperscript{10}

\textsuperscript{10} Because the position at each scheduled time was a posture rather than a location in extrinsic space, there would be no problem with bringing the unfelt arm to the posture that was originally scheduled. If the target posture had already been planned, there would be no need to sense the limb's current position.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure17.png}
\caption{Effect of number of stored postures on spatial accuracy and planning time. One hundred test reaches were simulated to yield each mean; the tests were administered immediately "after birth" with learning prevented during the test reaches; the targets were randomly situated within area A, C, or E; and the number of random, stored postures was 111, 333, or 666. As seen in the top panel, the mean spatial error before the first feedforward correction cycle generally declined with the number of stored postures, and, as seen in the bottom panel, the mean number of feedforward correction cycles needed to achieve a given level of spatial error (5 pixels) also declined with the number of stored postures.}
\end{figure}
Figure 18. Distance between the thumb and index finger (hand aperture, shown as short dashed lines), position of the hand (hand transport, shown as solid lines), and forward acceleration of the hand (bottom curves) in a trial in which a backward pull was applied to the arm with a force of 15 N in Panel A and with a force of 20 N in Panel B. The letter C indicates when the hand grasped the dowel being reached for. From "Remote Responses to Perturbation in Human Prehension," by P. Haggard and A. M. Wing, 1991, Neuroscience Letters, 122, p. 105. Reprinted by permission.

With respect to reaching, the way an object is grabbed depends on what will be done with it (Marteniuk, MacKenzie, Jeannerod, Athenes, & Dugas, 1987; Rosenbaum & Jorgensen, 1992; Rosenbaum et al., 1990; Rosenbaum, Vaughan, Jorgensen, Barnes, & Stewart, 1993). Similarly, the posture adopted in reaching for a target at a given spatial location depends on what spatial location must be reached afterward (Fischer et al., 1994), and the distribution of locations to which a hand-held stylus is brought in aiming for a target depends on the size and location of the next target to which the stylus will be brought (Sidaway et al., 1993).

It is possible to model such coarticulation effects with the Knowledge model. Suppose the actor needs to reach for one target after another. The unelaborated model would have the actor plan a target posture for the first target and reach for that target, and then plan a target posture for the second target and reach for that target. This method would suffice if the actor could pause at the first target while planning the reach to the second target; however, if the time spent at the first target had to be less than the time spent planning the reach to the second target, the second reach would have to be planned before the first reach was completed. One possibility would be to plan the second reach before or during performance of the first reach. If the second reach were planned, taking into account the target posture at the first target, the posture at the second target could be adaptively related to the first target posture. However, the first target posture would not change in anticipation of the second target posture unless there were complex interactions between the movement to Target Posture 1 and the ongoing planning of Target Posture 2.

A simpler way to ensure anticipation would be to plan the posture for the second spatial target and then to plan the posture for the first spatial target. The posture chosen for the second spatial target could be optimized with respect to the travel cost

from the starting posture and with respect to the posture’s spatial error from the second target. The posture chosen for the first target could be optimized with respect to three factors: the spatial error from the first target, the travel cost from the starting posture, and the travel cost from the first target posture to the second target posture. In effect, the planning system could answer the question, “What is the least costly detour to the first target on the way to the second target?” Finding an answer to this question could reduce the overall cost of movement for the first and second reaches, taken as a pair. Representative simulations based on this method are shown in Figure 19.

This way of modeling anticipation suggests a possible reason why coarticulation effects exist. Coarticulation helps reduce costs (in this case, travel costs). It is noteworthy, too, that this form of planning is hierarchical: The two postures are planned as a pair, and one is subordinate to the other (i.e., Target Posture 1 is subordinate to Target Posture 2). The fact that hierarchical planning economizes travel costs is appealing because there has been extensive support for the hypothesis that the serial ordering of behavior is hierarchically controlled (Lashley, 1951; Pew & Rosenbaum, 1988; Wright, 1990). However, the reason for this form of organization has been obscure. According to the Knowledge model, hierarchical control of series of movements emerges as a natural consequence of cost containment.

A final point about coarticulation is that it bears on the issue of controlling movements through target locations (so-called via points). If a movement is made through a desired location on the way to some other desired location, aspects of the movement must be planned ahead of time. A natural way to think about movements through via points is to suppose that a posture is planned for the via point; we call this a via posture. It is possible to propose a method for moving smoothly through a via posture on the way to a target posture based on a related method proposed by Flash and Henis (1991): Having selected a target posture and a via posture, initiate the movement from the starting posture to the via posture and, at some proportion, p, of the time to complete this movement, add the movement from the via posture to the target posture. The method is shown vectorially, along with a stick-figure simulation, in Figure 20.

An attractive feature of Flash and Henis’s (1991) method is that the final target posture is achieved no matter what the value of p. A somewhat less attractive feature is that the via posture is actually reached only when p equals 1, which is also the value of p for which the movement from the via posture to the target posture begins only after the movement from the start posture to the via posture has been completed. As p decreases from a maximum of 1 to a minimum of 0, there is more and more undershoot of the via posture until, when p equals 0, the undershoot is maximal. Thus, there is a trade-off between smoothness of movement, achieved by blending the two movements, and spatial accuracy. A possible way to cope with this problem is to aim for via locations beyond the true via point, by a distance inversely related to p.

### Posture Subspaces

In hypothesizing stored postures, an important question is how many postures should be stored. As seen earlier, the number of stored postures can be an important determinant of individual differences.

In the limit, the smallest number of values that needs to be stored for a given degree of freedom is two: the smallest and

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11 In fairness to Flash and Henis (1991), it should be noted that they did not devise their method to ensure passage through via points. Rather, they devised their method to account for data from experiments in which participants aimed for one visually signaled target and then had to aim for another target specified by a visual signal at another, unexpected location. The participants’ task was to get to the second target without necessarily having to pass through the first.

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![Figure 19](image-url) Coarticulation effects associated with selecting a posture for Spatial Target 1 after selecting a posture for Spatial Target 2 (left panel) in comparison with movements to target postures planned in isolation for Spatial Target 1 and Spatial Target 2 (right panel). Model parameters and other conventions are the same as in Figure 7.
largest values. As a result, the smallest number of stored postures needed for n degrees of freedom is $2^n$. For example, in the case of $n = 3$ degrees of freedom, the stored postures can be conceptualized as occupying all $2^3 = 8$ corners of a rectangular solid. Because the human body has about 100 mechanical degrees of freedom (Turvey, 1990b), at least $2^{100}$ stored postures would be needed for skillful movement. This number is unappealingly large, so the question arises of how it can be reduced.

A possible answer is to rely on posture subspaces. Suppose that some degrees of freedom are nested within others. For example, the hand (along with the fingers) might be nested within the wrist, so each small change of the angle of a finger need not be represented along with, say, a corresponding knee angle. The origin of the frame of reference for this subordinate posture space could "ride" the wrist. As the wrist moved, the posture space for the hand could shift with it, maintaining its orientation with respect to the forearm. This method of representation is often used in robotics (Craig, 1986) and has precedent in psychology. For our purposes, hierarchical organization of posture space would reduce the number of stored postures. For example, if the hand had $h$ degrees of freedom and the joints proximal to the hand had $q$ degrees of freedom, the minimal number of degrees of freedom of the entire body would be $2^q + 2^h$, rather than the larger number, $2^{q+h}$.

Moving Starting Postures

The Knowledge model assumes that target postures are planned with respect to starting postures. It is relatively straightforward to identify a starting posture when one is stationary, but what happens if one is moving and future movements must be planned?

In principle, the model can be extended to cases in which movements must be planned while other movements are under

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12 The minimal set of vectors needed to span a space of $n$ dimensions through linear combination is generally the $n$ basis vectors for the space. However, with the constraint that the weights for the vectors are non-negative and sum to 1, regions of the space cannot be reached with the usual $n$ basis vectors. For example, the upper right quadrant of an $x$-$y$ space can be reached only if the weights of the $x$ and $y$ vectors are both greater than $0.5$, which is forbidden if the $x$ and $y$ weights must sum to 1. This is why, in our system, the minimal set of stored posture vectors extends to the minima and maxima of the represented degrees of freedom.

13 The notion of posture subspaces has been implicit in previous discussions of sensorimotor control. The proposal by Flanders, Helms Tillery, and Soechting (1992) that arm movements are planned in a shoulder-centered coordinate frame is a well-known example of this concept, as is Jeannerod's proposal that arm transport and finger grip are controlled by two largely independent subsystems (see Jeannerod, in press). The idea of subordinate frames of reference has also been discussed in the study of visual perception, most notably by the Gestalt psychologists. The best-known example is Koffka's (1935) demonstration that a diamond is perceived as a diamond if it appears in a horizontally oriented rectangle (a rectangle none of whose edges are parallel to the edges of the diamond) but is perceived as a square if it appears in an obliquely oriented rectangle (a rectangle all of whose edges are parallel to the edges of the diamond).
way. All that is needed is to allow current or forthcoming postures to be treated as anticipated starting postures with respect to which later movements are planned. Future experiments can help test this hypothesis. For now, the key point is that the Knowledge model appears able to accommodate planning during motion as well as stability.

**Reaching for Moving Targets**

Actors do not always reach for stationary targets. They also reach for targets in motion. The Knowledge model can accommodate planning of reaches to moving objects, provided one allows that future locations of moving objects can be anticipated and that reaches can be directed to those locations and timed to end when the objects arrive at the anticipated interception sites. Evidence consistent with this pair of assumptions has been reported by Bairstow (1988), who observed corrections during the decelerative phase of movements directed to sites approached by moving targets when those moving targets were going to be intercepted.

In saying that the Knowledge model can apply to interception of moving objects, we are not claiming that it is the only model that can explain such behavior. Clearly, any model that can specify a location to which a contact point should be brought and that can govern the timing of movements to bring the contact point to that location before the object arrives can potentially explain interception behavior. A major challenge for any such model, however, is to determine where the interception site should be given that there are usually an infinite number of possible interception sites and given that, for each possible interception site, there are usually an infinite number of target postures that can bring the contact point to it (a kind of "double degrees of freedom" problem). A simple hypothesis from the Knowledge model is that the best interception site is the spatial location on the trajectory of the moving object to which the contact point is closest. Future research can test this hypothesis, and detailed measurements of movements during catching can provide a way of distinguishing predictions that are likely to emerge from the Knowledge model as opposed to other models.

**Maintaining Stability**

The Knowledge model can be extended to the maintenance of stability. The task of maintaining a posture can be understood in terms of aiming for an already-acquired spatial target (or set of spatial targets). Consider a person trying to hold his or her hand at a fixed spatial location when a load happens to displace the arm. Compensating for the displacement demands solution of the inverse kinematics problem, as in any reaching task. What distinguishes the compensation task from the typical reaching task, in which the starting posture is fixed and a new spatial target is identified, is that the starting posture changes and the spatial target remains the same. The computations assumed in the Knowledge model can be drawn on for compensation, however. Stored postures can be polled for their ability to bring the contact point back to the target, and a weighted sum can be taken to select the best target posture. The new target posture need not be the same as the old one.

This method will work effectively if the load that displaced the arm has disappeared by the time the arm returns to the target position, because then the posture that allowed the arm to remain at the target position before the load was imposed will allow the arm to remain there again. However, if the displacing load is still present, the arm will be displaced again, and the compensation will have to be repeated. The only way to avoid the need to repeat the compensation cycle is to specify muscle forces as well as muscle lengths in the target posture. The Knowledge model does not yet provide for specification of muscle forces, although it can be made to do so (as discussed later).

**Oscillatory Movements**

The suggestion has been made that oscillatory movements are somewhat more fundamental than discrete movements in the control of motor activity (Kelso, 1981). The Knowledge model does not regard back-and-forth movements as qualitatively different from single movements, however, in part because it does not run into the problem that there must be long delays between successive phases of cyclic movements when plans must be formulated for the successive phases of such movements. Instead, target postures and starting postures can simply trade roles: The target posture for movement \( n \) can become the starting posture for movement \( n + 1 \), and the starting posture for movement \( n \) can become the target posture for movement \( n + 1 \). Given this trading relation, the movement between the two postures can simply be run in reverse in succeeding cycles.

This method does more than allow for rapid alternation between successively repeated movements. It also gives rise to a result that has been taken to support the view that cyclic movements are more basic than discrete movements. Guiard (1993) observed that the muscular effort expended in individual movements is lower if those movements are performed in back-and-forth cycles than as one-shot responses. Such context effects led Guiard (1993) to claim that "the very concept of discrete movement is faulty in that half-cycles fail to capture the essence of real-life gestures, like pointing, grasping, and hitting" (p. 156). Guiard’s analysis failed to consider the possibility that the posture selected as the target for a movement can depend on where the movement originates (as outlined in the **Starting Position Effects** section) and that there can be cumulative, sequential

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14 An experimental procedure that can be used for this purpose would proceed in two stages. In the first stage, participants would reach for a target at a final spatial location, \( F_2 \), beginning at each of a number of starting locations, \( S_1, S_2, \ldots, S_n \), where those starting locations happened to occupy a typical hand path from \( S_1 \) to some other final target location, \( F_1 \). One would expect the posture adopted at \( F_2 \) to depend on the starting location, replicating previous starting-posture effects. In the experimental conditions, participants would initially reach for \( F_2 \), starting at \( S_1 \), along the way, however, a light would turn on at \( F_3 \), indicating that \( F_2 \) rather than \( F_1 \) should be the final target location. In the control conditions, a light would turn on at \( F_1 \) alone. The main question would be what final posture would be adopted at \( F_2 \). Is it the final posture previously shown to occur when participants started from \( S_1 \), or the final posture previously shown to occur when participants started from \( S_2 \), and so forth? If a clear answer could be found, one could infer that planning began from the inferred starting posture.
effects (as outlined in the Settling In section). The Knowledge model predicts, in fact, that if a person moves back and forth between two postures, the first target posture will be planned with respect to the starting posture, the second target posture will be planned with respect to the first target posture, the third target posture will be planned with respect to the second target posture, and so on. Simple switching of starting and target postures of the sort mentioned in the preceding paragraph will occur when the target postures adopted at either end of the cycle cease to vary significantly (i.e., when settling in has occurred) or when the actor is indifferent to travel costs. The Knowledge model predicts, then, that the kinematics of movements should differ in cyclic performance and in discrete performance, as Guiard (1993) observed, but not because the concept of discrete movements is faulty.

There is another way in which the Knowledge model’s emphasis on discrete movement planning can be preserved in the face of Guiard’s (1993) criticisms. Guiard suggested that the acceleration profiles of back-and-forth movements indicate that, during cyclic performance, movement \( n \) is initiated before movement \( n - 1 \) is completed. Such overlap of movements can be planned through a scheme such as the one proposed by Flash and Henn’s (1991; and as discussed earlier in the section titled Anticipatory Effects in Reaching).

Simulating Handwriting

One domain in which it is important to make smooth successive movements is handwriting. We have already extended the Knowledge model to simulate handwriting behavior. Preliminary results of the simulations were reported by Meulenbroek, Rosenbaum, Thomasson, and Loukopoulos (1993, in press).

Handwriting poses a challenge to the Knowledge model because the model predicts variations in movement curvature depending on movement direction (see Figure 13). In handwriting, however, the curvature of a stroke determines its identity (and therefore the identity of the letter to which it belongs); thus, writers must be able to control movement curvature. Another challenging feature of handwriting is related to the fact that the Knowledge model focuses on the planning of discrete reaching movements bounded by points of zero velocity. In handwriting, however, movements go on continuously; writers often produce long stroke sequences in which individual strokes may be bounded by points of nonzero minimum velocity. Despite these challenges, handwriting can be simulated in the Knowledge model. What is needed is a mechanism that allows the end effector to move through a spatial target (or via point) on its way to a subsequent target. The positions of the intermediate and final target relative to the starting position of the end effector (the pen tip in this simulation) determine the curvature of the movement path. By positing a via point mechanism, curvilinear trajectories can be modeled without violating the Knowledge model’s assumptions. The second extension of the model that allows for simulation of handwriting involves temporal overlap of movements (Flash & Henn, 1991), which we discussed earlier in connection with reaching to moving targets and reaching from moving starting postures. Overlapping small-scale movements allows one to simulate the quasi-continuous movements of writing behavior. Figure 21 shows an example of the writing of the cursive letter \( f \).

Generalizing the Model and the Concept of Tasks

Although the discussion so far has mainly concerned unimanual reaching in the sagittal plane, the Knowledge model can be generalized to other effectors, to movements in three spatial dimensions, and to dynamics as well as kinematics. Regarding dynamics, the model can be generalized so that posture representations include information about muscle forces as well as muscle lengths. Muscle forces and muscle lengths are independent, except at the extremes of each muscle’s length-force curve. The set of muscle forces acting at a joint defines the stiffness of the joint (i.e., its resistance to angular displacement), and, because the specification of muscle lengths is equivalent to the specification of joint angles, muscle forces and muscle lengths can be coded in a posture-based storage system by storing, along with the vector of angles for the joints’ angular degrees of freedom, the stiffnesses of those degrees of freedom. With information about stiffnesses and joint angles in each stored-posture record, the stored postures can be evaluated in essentially the same way that stored postures are evaluated in Knowledge II: Those stored postures that achieve necessary joint angles and necessary stiffnesses can be given high weights, whereas those that do not can be given low weights, and a weighted sum of the stored postures can be found to specify a target posture defined with respect to muscle lengths and forces. In this scheme, the travel cost can represent the cost of shifting from one set of stiffnesses to another, along with the cost of shifting from one set of joint angles to another. Greater emphasis can be placed on satisfying joint-angle requirements or on satisfying stiffness requirements, just as greater emphasis can be placed on satisfying spatial error costs or travel costs. More research will be needed, of course, to determine how well this system can work. It is clearly imperative to extend the Knowledge model to dynamics, because real actors must take forces into account to compensate for loads, manipulate objects, maintain balance, and so forth.

The foregoing discussion highlights the Knowledge model’s potential for application to a wide range of tasks and costs. Virtually any motor act can be conceptualized as a transition from one posture to another; it does not matter whether the posture involves the arms, fingers, speech apparatus, or legs (to name just some effectors). Moreover, any kinds of costs can be dealt with. For example, it would be natural to add a balance cost, representing the importance of keeping the center of gravity of the body over the hip (in sitting) or over the feet (in standing). It would also be possible to add a cost associated with departing from the middle of the range of motion for each joint. People prefer to finish movements at or near the middle of the range of joint motion (Cruse et al., 1993), especially at the end of a series of movements needed to complete a task (Rosenbaum & Jorgensen, 1992, Rosenbaum et al., 1990, 1993). Extending the model to movement in three-dimensional space is also straightforward. The spatial error cost for a stored posture would reflect the distance between the contact point and the target location in three-dimensional space. More interestingly, a task goal
could be described in terms of a set of spatial target locations, each associated with a distinct contact point, rather than a single spatial target location associated with just one contact point. With multiple spatial target locations and contact points, it would be possible to represent a task such as grasping a cup in terms of a place on one side of the cup that should be contacted with the tip of the thumb, a place on the other side of the cup that should be contacted with the tip of the index finger, and so on. Forces for grasping could be specified either directly or indirectly in terms of virtual spatial target locations within the cup to which the contact points should be directed (Bizzi et al., 1992); the greater the depths of the virtual spatial locations, the greater the forces that would be exerted when the cup is grasped.

Because the Knowledge model is abstract, it need not be restricted to reaching. In reaching, the most relevant costs pertain to spatial errors and factors related to rotational kinetic energy. However, other costs can be postulated for whatever effectors are most relevant. In the case of speech production, for example, it is crucial to select an oral posture that produces a desired sound; in the case of moving the eyes, it is essential to select an orbital position that permits the stimulus to fall on the fovea; in the case of hitting a tennis ball, it is essential to find a posture that permits the racket to strike the ball on the way to some other position; and, in the case of standing, it is essential to find a posture that keeps the center of gravity above the base of support. Noting such examples, we can describe the Knowledge model in more general terms than we have done so far: According to the Knowledge model, the goal of planning is to take a weighted sum of stored postures such that the total cost of each stored posture is a weighted sum of all the costs relevant to the task to be performed. Given this way of thinking about planning, a task can be conceptualized as a vector of salience values...
for all factors potentially relevant to what one may have to do.
In speech, for example, coming close to a target sound is important, but in reaching, generating a particular sound is generally unimportant. What distinguishes the task of speaking from the task of reaching is, among other things, the importance of achieving target sounds. Thinking of tasks in this way, one can rewrite the total-cost equation of the Knowledge model (Equation 7) as follows:

$$T_p = \sum_{c=1}^{u} w_p C_{c p} \quad (16)$$

where $T_p$ denotes the total cost for posture $p$ and $w_p$ denotes the weight assigned to the $c$th of the 1, 2, \ldots, $u$ costs, including cost $C_{c p}$ for posture $p$. Equation 16 implies that the same set of postures can be recruited for all tasks and that switching from one task to another (e.g., switching from reaching to speaking) is no different mathematically from adjusting weights for particular costs within particular task contexts (e.g., varying the spatial error weight and travel-cost weight for sagittal-plane reaching).

### Relations to Other Models of Motor Control

In this section, we briefly compare the Knowledge model to two other models that have been developed to address the inverse kinematics problem and related issues. We limit ourselves to models not discussed in earlier presentations of the Knowledge model. In Rosenbaum et al. (1993a), we evaluated models of Hinton (1984), Jordan and Rumelhart (1990), and Rosenbaum et al. (1991).

**Bizzi et al. (1992)**

The ideas of Bizzi and his colleagues have already been mentioned here. These investigators championed the concept, which originated with Asatryan and Feldman (1965) and Crossman and Goodeve (1983), that movements are defined by series of equilibrium points. Two aspects of the theory of Bizzi et al. deserve comment here. One pertains to their framework for understanding stimulus-response translation:

We summarize briefly the transformations that are thought to occur when a sensory stimulus (such as an object to be reached) appears in the environment. The first step in carrying out a reaching task involves a transformation [that] represents the location of [the] object with respect to the body and the head. \ldots The second step involves planning of the direction of hand motion and presumably its velocity and amplitude. (Bizzi et al., 1992, p. 603)

The first stage in the Knowledge model is the same as the first stage assumed by Bizzi et al. (locating the object to be reached with respect to the body), but the second stage is different. In the Knowledge model, movement directions are not planned after objects are located; in fact, they are never planned explicitly. Instead, movement directions emerge from selection of target postures vis-à-vis starting postures. The second stage of perceptual-motor transformation in the Knowledge model is selection of a target posture rather than selection of a movement direction. An advantage of this approach is that hand paths do not have an invariant shape, contrary to what was first supposed (Morasso, 1981) and taken by Bizzi et al. to be a signal property of motor performance. Instead, hand paths depend on where the hand must be directed in space (see the earlier discussion of this topic). If hand-path directions depend on where the hand will go, parsimony suggests that selection of the target posture is all that needs to be specified directly.

The second noteworthy aspect of Bizzi's theory concerns force production. Bizzi et al. (1992) argued that aiming for spatial target locations provides a way of governing contact forces. They proposed that aiming for a spatial target inside a surface permits the actor to generate forces on the surface when the body contacts it. The greater the distance between the surface and the virtual target, the greater the force to be produced, within limits. We find this idea appealing, as indicated earlier in our discussion of grasping cups, and we plan to pursue it in future work with the Knowledge model. In fact, Vaughan et al. (1993) showed that, when participants tapped the forefinger against a surface with varying impulses of collision, the contributions of the finger, hand, and forearm could be related to virtual spatial targets located inside the struck surface; the calculated depth of those spatial targets was proportional to the impact of collision.

**Bullock et al. (1993)**

Bullock et al. (1993) developed a theory aimed at solving the same set of problems as the Knowledge model. To set the stage for their theory, they explicitly compared their perspective with two others. One they called "motor trajectory formation." Here, muscle lengths are mapped to spatial target locations. The problem with this approach, according to Bullock et al., is that it does not allow for specification of spatial trajectories, which may be crucial in some circumstances (e.g., drawing desired figures or avoiding obstacles). One might comment that this problem could be circumvented by selecting a series of spatial via points to which muscle lengths are mapped successively, as in the approach of Bizzi et al. (1992); the path-selection problem might then be avoided. This comment notwithstanding, Bullock et al. were correct to question the usefulness of mapping muscle lengths to spatial locations because the effect of contracting a muscle is highly dependent on the initial configuration of the limb (Enoica, 1988). Unless such interactions are known and can be compensated for, specification of muscle lengths is unlikely to be the sole means of motion planning.

The second approach that Bullock et al. (1993) considered was one they called "spatial trajectory formation." The essence of this approach is mapping joint-angle configurations (postures) to spatial targets. Of course, this is just the approach we are advocating. The main problem with spatial trajectory formation, according to Bullock et al., is that averaging two joint configurations does not necessarily yield a new joint configuration that maps to a desired spatial endpoint because of the nonlinearity of forward kinematics. Another difficulty is that blind reaching with a tool should, according to Bullock et al., be hard or impossible, and compensation for changes in joint-angle mobility should take too long to be practical.

We have several reactions to these criticisms. First, we have shown that a system that maps postures to spatial targets can
plan reaches with a tool, provided it can compute forward kinematics (see Figure 8). Second, such a system can compensate, within one planning cycle, for a change in the mobility of the joints (see Figure 11). Thus, as we observed earlier (see Footnote 7), planning based on postures can compensate more quickly than Bullock et al. (1993) supposed. Third, although we agree with Bullock et al. that the nonlinearity of forward kinematics makes it impossible to ensure spatial target attainment given averaging of posture vectors, endpoint lacunas of the sort that would be expected if such averaging occurred have, in fact, been observed (see the section titled *Error Dead Zones and Motor Scotomas*). Granting the undesirable character of such lacunas, we have devised a computational method for offsetting the inaccuracy that accrues from posture averaging (feedforward correction). Although some might argue that this method is ad hoc, feedforward correction has the virtue of predicting planning times (see Figure 16). Moreover, if feedforward correction occurs while motion is in progress, it predicts homing in on spatial target locations, as in fact occurs (Meyer et al., 1990).

A final observation about Bullock et al.’s (1993) criticism of posture-to-location mapping is that one can question the equation of this form of mapping with averaging of joint configurations (postures). One could map postures to locations through other means. For example, one could use gradient descent to identify a desirable target posture. The gradient could be defined according to spatial error cost and travel cost, as well as other possible costs. Such an approach can work effectively (Engelbrecht & Rosenbaum, 1994) and so may obviate the difficulty of posture averaging.

The third approach to planning that Bullock et al. (1993) considered was the one they embraced. They called it “spatial trajectory with direction mapping.” Here, the planning primitives are joint-angle velocity vectors. Linearly combining these vectors (taking their weighted sum) has the desirable property that the resultant vector points in the same direction as the component vectors, provided the component vectors all point in the same spatial direction. If the component vectors do not all point in the same spatial direction, the resultant vector points in a spatial direction that is a linear combination of the component vectors. Bullock et al. argued that this sort of predictability offers prima facie evidence for reliance on joint-angle velocity vectors. In their system, the actor learns to associate joint-angle vectors with spatial directions of movement. The vectors are combined as necessary to move the hand to the target. Simulations conducted by Bullock et al. showed that their system yielded realistic movement patterns, compensated for freezing of joints, allowed for pointing with a hand-held tool, and permitted adaptive responses to perturbations of limb positions. In addition, Bullock et al. noted that there are correspondences between their model’s assumed computations and physiological properties of neurons related to motor control (e.g., directional tuning of cells in the motor cortex).

The model of Bullock et al. (1993)—what they called the DIRECT model—is impressive. Our question is whether it is adequate. The DIRECT model makes a prediction that appears to be violated by existing data. In the DIRECT model, spatial directions on either side of the midsagittal plane are coded with respect to the same frame of reference. As a result, rightward motions of the two hands have the same spatial direction, and leftward motions of the two hands have the same spatial direction. Now consider bimanual movements. It is well known that movements of the two hands are coupled (Heuer, in press; Turvey, 1990b). One would expect that the coupling would be defined with respect to factors that the motor system uses for movement planning and control. Because, according to Bullock et al., motions are planned with respect to extrinsic spatial directions, one would expect that if the two hands were coupled, they would be biased to move in the same spatial direction rather than in opposite extrinsic spatial directions. In fact, the coupling of the hands is the opposite. As first observed by Cohen (1970) and then studied in detail by Kelso and his colleagues (e.g., Schöner & Kelso, 1988), simultaneous oscillation of the two index fingers is easier if the two fingers flex together and extend together than if they flex and extend simultaneously. When the two index fingers make homologous movements (both flexing or both extending), their extrinsic directions of movement are opposite. Therefore, the DIRECT model seems to make the wrong prediction about performance in this situation. If it had to predict what sort of coupling is easier, it would predict that nonhomologous movements (i.e., movements made in the same extrinsic direction) should be easier than homologous movements (i.e., movements made in opposite extrinsic directions). The Knowledge model predicts that if there is a linkage between the hands, there should be a tendency for one hand’s posture to be similar to the other hand’s posture, which is just what occurs.

A hint that the two hands are, in fact, biased to share postures comes from a study conducted by Paillard and Brouchon (1968) in which participants were able to match the position of one hand with the other even if neither hand could be seen and even if the distances to be covered by the two hands did not match. As noted by Welford (1968), this result suggests that “the accuracy of . . . movements depends on some absolute appreciation of end position rather than of distance moved” (p. 160). One might say that participants in the experiment of Paillard and Brouchon learned external spatial locations (or external spatial heights) rather than postures. We doubt that this interpretation is correct, however, and can propose a simple experiment to show that it is not. Repeat Paillard and Brouchon’s study with the participant bent sideways. The accuracy of matching the position of one hand with the other should be reduced because the joint angles of the two arms differ.

Final Remarks

In this article, we have presented a theory concerning the planning of reaching movements. The central idea is that stored postures are used to select reaching movements and that costs of possible postures and postural transitions are taken into account in the selection process. The immediate aim of the theory is to provide a means of planning one-handed reaches in the sagittal plane. The long-term aim is to provide a framework for understanding the generation of any type of physical action.
Additional Sources of Support

We have been encouraged by the wide range of phenomena that the theory can explain, although we recognize that more data are needed to evaluate the theory's predictions and to help choose among theoretical options that remain. We note as well that there are published results bearing on the theory that we have not reviewed here. In particular, there are data concerning speech production that can be used to evaluate the hypothesis that stored articulatory postures are used to plan speech gestures, there are data concerning locomotion that can be used to evaluate the hypothesis that stored leg postures are used to plan walking movements, and so on. These data are best considered when the model is directly applied to these other activities. (Some readers may be interested to know that we have generated an animation of a walking stick figure based on the premise that walking can be viewed as a series of transitions between the same two whole-body postures.)

Neurophysiology and neurology. There are also neurophysiological and neurological (clinical) data that bear on our theory, but we are hesitant to review these data in detail because of the complexity of the literature and our relative lack of expertise with it. We do wish to highlight a few of these findings, however, because they seem to provide particularly striking support for our viewpoint. First, as is well known, and as we mentioned earlier in this article, neurons in a number of brain regions have broad tuning curves. For example, motor-cortex neurons of the monkey brain fire preferentially to different directions of forthcoming movement. As Georgopoulos and his colleagues have shown (Georgopoulos, 1990, 1991), the actual direction of movement is well predicted by a weighted sum of the individual directions represented by these neurons, where the weights are conveyed by the neurons' activity levels. This outcome is consistent with the Gaussian averaging component of the Knowledge model. Our procedure for finding weights can be viewed as a method for determining activation levels of processing elements, and our procedure for taking a weighted sum can be viewed as a method for achieving population coding of those elements.

A number of clinical syndromes also fit with our theory. We predict that if there is damage to the posture store, with an accompanying loss of stored postures, there should be reduced accuracy for reaches to spatial regions that principally use those postures. Lesions to the frontal and parietal areas of the brain have just this effect (Jeannerod, in press). Moreover, and more subtly, biases in performed directions of motion can be influenced by lesioning cortical areas (reviewed by Jeannerod, in press), as if the postures stored in those regions have less influence than normal.

Commercial animation. Another domain in which we have found support for our theory is one seldom cited in the cognitive and neuroscience literature. This is commercial animation. When Walt Disney and his colleagues first explored methods of animation, the technique they found most efficient was key framing—creating key poses and then interpolating poses between them (Blair, 1949). This technique proved to be much easier than "straight-ahead action," in which each frame drawn was the next one to be viewed. With key framing, a high-level artist draws the key poses of characters and objects, and then lower level artists fill in the gaps. Key framing is now used widely in computer graphics (Lasseter, 1987). An advantage of key framing is that it lends itself to top-down (hierarchical) production of event sequences: The most important events are drawn first, less important events are interposed between the most important events, still less important events are interposed between the second most important ones, and so on. Key framing also appears to be the method of choice when exact poses and precise timing are required (Lasseter, 1987).

The similarities between key framing and our system are obvious. Although we cannot conclude from the success of key framing that the nervous system uses the same method, it is reasonable to think that challenges faced by commercial animators are similar to those faced by biological animators. If key framing has proven most efficient when carried out by teams of individuals animating cartoon figures, it may be most efficient when carried out by teams of cognitive or neural modules animating real figures.

Analysis by Synthesis

The last point we wish to make concerns the method we have adopted for studying movement planning and the promise our model holds for other performance domains outside motor control. The method we have adopted to analyze the system we wish to understand is to synthesize its. This approach forces one to confront all of the issues that ultimately must be dealt with by the system under study. Narrowing the scope of the problem is one way to make this task tractable. Still, even with a more limited domain of application (kinematics rather than dynamics, for example, or one-handed sagittal-plane reaching rather than a broader array of movement types), one must make a set of clear assumptions to get the job done. A virtue of this style of research is that, with the theory fully developed, one has a system that may have practical value. Commercial animators or roboticists may find our model useful, for example. Another virtue of the analysis-by-synthesis approach is that, with all the assumptions spelled out explicitly, the theory can become a foil for other researchers. Even if the Knowledge model only motivates others to develop simpler, more powerful theories, we will consider our efforts worthwhile.

Whatever the ultimate theory of movement planning will look like, we believe that it will have a number of features in common with the Knowledge model. Indeed, we hold out hope that it may extend to other domains outside motor control. Recall that the Knowledge model was developed to solve the degrees of freedom problem. This problem arises whenever more degrees of freedom exist in a system responsible for carrying out a task than in the ostensive description of the task. The degrees of freedom problem arises in vision (assigning a three-dimensional structure to a two-dimensional retinal image), in credit assignment tasks (determining which aspects of a system contribute to its success or failure), and in any situation in which one must find a solution to a problem for which alternative solutions exist. Given how widespread the degrees of freedom problem is, it is possible that solutions to it in one domain may generalize to others.
The essence of the Knowledge model is that specialized modules become activated in relation to their fit to task demands, and the global response of the system is based on a weighted sum of the modules. Theories of this sort have been developed in the fields of vision (Ullman, 1990) and memory (Hintzman, 1988). One reason why this type of theory is appealing is that it minimizes the role of an omniscient executive. Only when such a homunculus has been eliminated can one claim to have solved the problem of cognition. The Knowledge model is one approach to this difficult challenge. Although the model has been applied to physical action, and many issues remain to be resolved for its full development, it holds promise as a way to understand how intelligence is expressed in a wide set of domains.

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