

# Timing of Two-Handed Rhythmic Performance<sup>a,b</sup>

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## INTRODUCTION

In this paper we present a simple stochastic model for the study of timing of skilled performance. The model is based on the notion that temporal variability and serial dependence observed in behavior sequences depend on the precision of a hypothetical central timing system as well as on the temporal jitter generated by the executing motor system. Within the theoretical framework of such a model it is possible to determine, for example, how much of the temporal variability in skilled performance is generated centrally, and how much is due to the precision lost during the execution of the motor program by the motor system. This general approach has been used first by Wing and Kristofferson<sup>1</sup> in their study of timing in simple tapping situations. We have proposed extensions of the basic model to repetitive rhythmic tapping; our goal was to determine whether rhythmic performance is controlled by hierarchically organized timing mechanisms, as contrasted to simple serial mechanisms.<sup>2</sup> Analyses of actual musical performance in terms of such a model have been reported by Shaffer.<sup>3</sup>

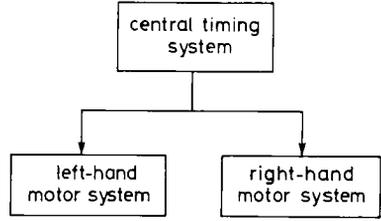
A major problem with these models is that they rest on rather strong assumptions which are not easily accessible to empirical test. Of course, the conclusions reached from applications of the models remain dubious if the validity of the underlying assumptions can be questioned. In the following, we show how to overcome some of these problems by extending the model to synchronous two-handed performance. The model we propose rests on weaker and testable assumptions; moreover, it permits rather strong inferences about the structure of the postulated timing system given that the model's validity can be established. It should be noted that several independent investigators have discovered recently how powerful the analysis of two-handed performance is in the study of timing and motor control.<sup>3,4-6</sup>

In this paper we focus on the properties of the central timing structures underlying synchronous rhythm production; in a companion paper<sup>7</sup> we investigate the statistical aspects of the motor system in more detail. The outline of the paper is as follows. First, we present the model and derive a testable prediction which is checked on data from an experiment involving two-handed rhythmic tapping. The model is then used as a

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FIGURE 1. General model for the timing of synchronous two-handed rhythm production.



theoretical framework within which we try to distinguish between serial and hierarchical timing structures.

**THEORETICAL FRAMEWORK**

Our theoretical framework for the study of timing of synchronous two-handed rhythmic performance is shown in FIGURE 1. We assume a single central timing system generating motor commands which are sent to both the left-hand and the right-hand motor subsystem simultaneously. FIGURE 2 shows how these three systems operate in time when a repeating rhythm is beat bimanually. The tick marks on the time axis in the middle row indicate the (nonobservable) time points when the successive central commands occur. Each command triggers a response in both the left-hand and the right-hand system; however, observable responses occur only with some delay after their corresponding central commands. We assume that these *motor delays* are random variables; we denote them by  $L_n$  and  $R_n$ , where  $L_n$  ( $R_n$ ) is the time difference between the occurrence of the  $n$ th left-hand (right-hand) response and the command that triggered it. Similarly, we assume the *timer intervals* between successive central commands as random variables denoted by  $T_n$ ,  $n = 1, 2, \dots$ , where  $T_n$  is the interval

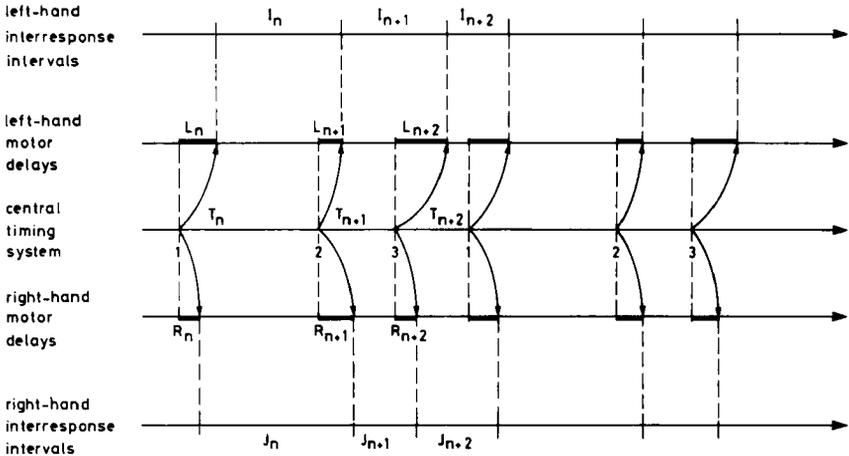


FIGURE 2. Hypothetical timer intervals ( $T_n$ ) and motor delays ( $L_n, R_n$ ) and their relation to the observable inter-response intervals of the left-hand ( $I_n$ ) and right-hand ( $J_n$ ) response sequences. A rhythm consisting of three notes (1, 2, 3) is assumed to be produced repeatedly.

bounded by the  $n$ th and the  $(n + 1)$ th command. The following assumptions are made:

- A1: Common timing system.* The same central timing commands control both motor subsystems.
- A2: Independence between systems.* The timer intervals,  $T_n$ , the left-hand and the right-hand motor delays,  $L_n$  and  $R_n$ , respectively, are independent of each other; for all  $i, j, k > 0$ ,  $\text{cov}(T_i, L_j) = \text{cov}(T_i, R_k) = \text{cov}(L_j, R_k) = 0$ .
- A3: Order preservation.* The order of responses set up by the central timing system is not altered by the motor systems, that is,  $P(T_n + L_{n+1} - L_n > 0) = 1$  and  $P(T_n + R_{n+1} - R_n > 0) = 1$  for all  $n$ .

Some comment on the assumptions is in order.

The assumption of a common timing system which controls both hands (*A1*), though plausible, might be violated when a performer notes that one hand consistently lags behind the other and tries to compensate for this by triggering the early hand only after some delay, that is, by using separate commands for the two hands. We try to minimize the likelihood that this happens in our experiments by providing auditory feedback only about the response that occurs first.

Independence between the timer and the motor systems (assumption *A2*) might break down at fast tapping rates. If successive motor delays within a hand are correlated and if the degree of correlation depends on how closely in time two responses follow each other, then variability of the timer intervals,  $T_n$ , will affect the dependence structure of the motor delays and therefore violate the independence assumption. For tapping at medium or slow tempos, however, this problem should be negligible.

The order preservation assumption (*A3*) holds if the variability of the difference between successive motor delays, for example,  $\text{var}(L_{n+1} - L_n)$ , is small relative to the corresponding mean interresponse intervals,  $E(T_n + L_{n+1} - L_n)$ . From previous results,<sup>1,2</sup> *A3* is almost certainly true.

It should be noted that independence is assumed only *between* systems, whereas the random variables *within* any system may be arbitrarily correlated. This stands in marked contrast to previous work on timing by us and others. With the exception of Wing,<sup>8</sup> successive motor delays were assumed to be independent.<sup>1-3,6,9</sup> Similarly, the intervals generated by the timing system were either assumed independent<sup>1,6,8,9</sup> or decomposable into independent random variables.<sup>2,3</sup> The main advantage of studying two-handed rhythmic performance is that it permits testing of the independence between system as well as determination of the actual amount of dependence existing within the timer intervals and the motor delays rather than taking independence between and within systems for granted.

If assumptions *A1*–*A3* hold, the postulated timer and motor-delay random variables can be related to the observable interresponse intervals (IRIs). Let  $I_n$  denote the IRI bounded by the  $n$ th and the  $(n + 1)$ th left-hand responses; correspondingly,  $J_n$  denotes the  $n$ th right-hand IRI. FIGURE 2 illustrates these definitions.

The left-hand and right-hand IRIs are tied to the timer intervals,  $T_n$ , and the motor delays,  $L_n$  and  $R_n$ , by the basic equations

$$I_n = T_n + L_{n+1} - L_n, \quad (1a)$$

$$J_n = T_n + R_{n+1} - R_n, \quad n > 0. \quad (1b)$$

Equation 1 can be verified from FIGURE 2; its validity follows from assumptions *A1* and *A3*.

Equation 1 provides a simple tool for examining the timing system, and for

separating its contributions to the data from those of the two motor systems. By analyzing the covariation between left-hand and right-hand IRIs, it is possible to infer the statistical properties of the sequence of intervals produced by the timing system,  $\{T_n\}$ , and thus, potentially, its structure. The intuitive idea behind our analysis is as follows:

Our primary interest is on the stochastic process,  $\{T_n\}$ , generated by the timing system which is accessible to observation only up to some "temporal noise" that is added by the motor systems. However, the left-hand and right-hand IRI sequences,  $\{I_n\}$  and  $\{J_n\}$ , respectively, provide an opportunity to observe the same process  $\{T_n\}$  twice, once after the left-hand and once after the right-hand motor system has operated on it, adding the independent noise sources  $\{L_{n+1} - L_n\}$  and  $\{R_{n+1} - R_n\}$ , respectively. By comparing the left-hand with the right-hand IRI sequences, we can recover what they have in common, namely, the intervals  $\{T_n\}$  generated by the timing system (see Equation 1).

The statistical analysis of the model turns out to be particularly simple if the dependence relations between the IRIs are described in terms of covariances. For random variables  $X$  and  $Y$ , the covariance is defined as  $\text{cov}(X, Y) \equiv E[(X - \mu_x)(Y - \mu_y)]$ , where  $\mu_x$  and  $\mu_y$  are the expectations of  $X$  and  $Y$ , respectively.<sup>c</sup> The amount of dependence that exists between any pair of IRIs from the left-hand and the right-hand sequences is found as

$$\text{cov}(I_m, J_n) = \text{cov}(T_m + L_{m+1} - L_m, T_n + R_{n+1} - R_n)$$

(by Equation 1)

$$\begin{aligned} &= \text{cov}(T_m, T_n) \\ &\quad + \text{cov}(T_m, R_{n+1} - R_n) \\ &\quad + \text{cov}(L_{m+1} - L_m, T_n) \\ &\quad + \text{cov}(L_{m+1} - L_m, R_{n+1} - R_n) \end{aligned}$$

(by the distributivity of covariances; for example, Vorberg<sup>10</sup>)

$$= \text{cov}(T_m, T_n)$$

(by the independence assumption A2).

Thus, if the assumptions of our model hold, the covariance between any pair of left-hand and right-hand IRIs equals that between the corresponding timer intervals, but is independent of the properties of the motor delays.

Two immediate consequences of this result should be noted: First, since  $\text{var}(T_m) = \text{cov}(T_m, T_m)$ , the variance of an interval produced by the timing system,  $T_m$ , can be determined from the covariance between the corresponding left-hand and right-hand IRIs, that is,  $\text{var}(T_m) = \text{cov}(I_m, J_m)$ . Second, since covariances are symmetric in the variables (for example,  $\text{cov}(T_m, T_n) = \text{cov}(T_n, T_m)$ ), the dependence between any two IRIs is the same when the role of the hands is interchanged, that is,  $\text{cov}(I_m, J_n) = \text{cov}(T_m, T_n) = \text{cov}(T_n, T_m) = \text{cov}(I_n, J_m)$ .

We summarize these results:

$$\text{cov}(T_m, T_n) = \text{cov}(I_m, J_n) = \text{cov}(I_n, J_m), \tag{2a}$$

$$\text{var}(T_m) = \text{cov}(I_m, J_m). \tag{2b}$$

<sup>c</sup>The covariance equals the nonnormed product-moment correlation, that is,  $\text{cov}(X, Y) = \rho_{xy}\sigma_x\sigma_y$ .

Equation 2 can be used as a way to test the model empirically as well as a means of inferring the stochastic structure of the timing system from the IRIs, assuming the model to be valid. In the subsequent section, TEST OF THE MODEL, we examine whether the symmetry prediction (Equation 2a) holds for the left-hand and right-hand IRI sequences observed in two-handed rhythm production. Later sections then focus on the temporal variability and the dependence structure of the intervals produced by the timing system as revealed by the covariances between left-hand and right-hand IRIs.

The model as sketched so far is testable on two-handed tapping data without additional assumptions. Since we apply it to the production of repeating rhythms it seems natural, however, to add the assumption that the sequences of timer intervals,  $\{T_n\}$ , and motor delays,  $\{L_n\}$  and  $\{R_n\}$ , are periodic in the means, variances and covariances, where the period depends on the number of notes per rhythmic cycle.

*A4: Periodicity.* For repeated rhythmic cycles consisting of  $c$  notes, the sequences  $\{T_n\}$ ,  $\{L_n\}$ , and  $\{R_n\}$  are periodic in the means and covariances with period  $c$ . That is, for all  $m, n \geq 1$ ,  $E(T_n) = E(T_{n+c})$ , and  $\text{cov}(T_m, T_n) = \text{cov}(T_{m+c}, T_{n+c})$ , and analogously for  $\{L_n\}$  and  $\{R_n\}$ .

Since  $\text{var}(T_m) = \text{cov}(T_m, T_m)$ , *A4* implies periodicity in the variances as well. Note that the assumption allows the means, variances and covariances of the timer intervals and the motor delays to differ for the different responses within a cycle, thus relaxing the simplifying assumption of Vorberg and Hambuch<sup>2</sup> of constant motor-delay variances. Obviously, *A4* implies periodicity of the IRI sequences,  $\{I_n\}$  and  $\{J_n\}$ .

Periodicity is not an essential assumption for the tests of the general model reported in the following section; there, it is used for economical reasons only, justifying the estimation of variances and covariances from the corresponding serial statistics computed across IRI sequences of repeating rhythmic cycles. Periodicity will become important, however, when we examine the statistical structure of the intervals  $\{T_n\}$  generated by the timing system.

## EXPERIMENTAL METHOD<sup>d</sup>

Our experimental paradigm is patterned after the continuation procedure introduced by Wing and Kristofferson<sup>1</sup> (see Figure 3). Each trial starts with a computer-generated rhythm auditorily presented to the subject via headphones. The rhythm keeps repeating at a fixed rate until the subject starts tapping the rhythm on Morse keys; the subject is to respond in synchrony with the computer-generated tones. This synchronization phase lasts for four repetitions of the rhythm, called *cycles* hereafter. Then the tones stop, and the subject is to continue tapping the rhythm at the given rate for 16 unaccompanied cycles. The end of the continuation phase is signaled to the subject by a tone.

Four different rhythms were employed (FIG. 4) abbreviated by 2-4-2, 2-1-3-2, 3-1-2-2, and 2-2-2-2, where the numbers given indicate the ratios of the successive time intervals between the notes of the rhythm. The tempo was kept fixed at 175 msec per eighth note. Therefore, the actual time intervals of the rhythms are obtained by multiplying the number code above by 350 msec; for example, the successive notes in rhythm 3-1-2-2 were separated by 525, 175, 350 and 350 msec.

<sup>d</sup>A more detailed description of the method is given in a companion paper.<sup>7</sup> The experiment contained one-handed as well as two-handed conditions; data from the latter only are reported here.

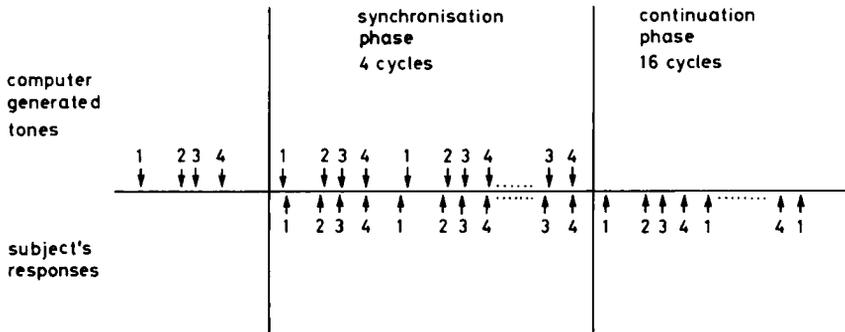


FIGURE 3. Schematic of an experimental trial; the four notes of the rhythm to be produced are denoted by 1, 2, 3, 4, 1, . . . .

Subjects were to beat the rhythms with both hands on two separate Morse keys on a table in front of them. They were instructed to produce the rhythm as precisely as possible, trying to keep the indicated tempo as well as close synchrony between the hands.

The tone sequences generated by the computer consisted of 50-msec sinusoids at a comfortable listening level; they were of equal pitch (400 Hz) except for the first tone of each cycle, which was two semitones up (449 Hz) to mark the cycle onset. Subjects were provided auditory feedback about their responses; immediately after being registered, taps elicited 50-msec tones an octave above the corresponding computer-produced tones. In order to prevent subjects from knowing how precisely they achieved synchronization of the hands, feedback tones were elicited only by the hand registered first.

The data from one professional musician and four amateur musicians with at least 6 years of instrumental practice will be reported. Subjects were screened from a preliminary sample of 26 persons. After two to five 1-hr training sessions, they served for 10 sessions in the main experiment. Precision of performance was constantly

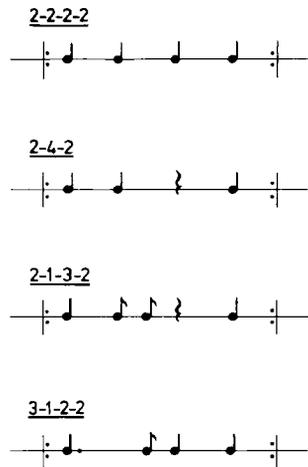


FIGURE 4. The four different rhythms employed in the experiment.

monitored during the experiment; any response sequence that deviated too much from the prescribed rhythms in terms of average tempo or the rhythmic pattern produced was to be redone immediately. During training, these criteria were gradually increased; the main experiment started when the final criteria were violated on no more than 5% of the trials. In order to keep subjects motivated, they were given visual feedback about their performance on a CRT that graphically indicated both the duration of the individual rhythmic cycles as well as the mean interresponse times within the rhythm averaged across the different repetitions. Information on whether a sequence was to be repeated was also presented via the CRT.

Each session consisted of two blocks with each of the conditions occurring twice per block. The different conditions were presented in random order except for the constraint that the two replications per condition followed each other immediately.

The data analysis of the IRI sequences  $\{I_n\}$  and  $\{J_n\}$  was performed separately for each subject. First, individual sequences were checked for "stationarity" by analyzing the durations of successive cycles (that is,  $I_1 + \dots + I_c, I_{c+1} + \dots + I_{2c}, I_{2c+1} + \dots + I_{3c}$ , and so on) for linear trend. Any sequence was eliminated for which the corresponding  $t$  statistic exceeded the critical value at  $\alpha = .25$ . Overall, 13.8% of the sequences were discarded due to nonstationarity.

Variances and covariances were estimated from the corresponding serial statistics for each pair of sequences,  $\{I_n\}, \{J_n\}$ , which were then averaged across all replications produced by a subject under the given experimental condition.<sup>e</sup> Standard errors of the statistics were estimated from their standard deviations across replications. All statistics are based on between 26 and 38 pairs of 16-cycle IRI sequences (average 33.4) per subject; this amounts to a total of about 2670 pairs of IRI per condition, on which each covariance estimate is based.

### TEST OF THE MODEL

Elsewhere<sup>7</sup> we have reported tests of the assumed independence between the timing and motor systems and with the predictions of the variances of the asynchronies between the two hands. In these tests, the model succeeded quite well, giving support for the general theoretical framework. Here, we examine the validity of Equation 2a as a prerequisite of our more detailed analysis of the timing system.

Consider the covariances between interresponse times of the left-hand,  $I_m$ , and the right-hand,  $J_n$ . For any  $m, n > 0$ , the covariance between them depends only on the statistical structure of the central timer and not on those of the motor systems, since  $\text{cov}(I_m, J_n) = \text{cov}(T_m, T_n)$  by independence (Equation 2).

As was seen above, an important feature of the model is the symmetry in the dependence of the left-hand and the right-hand IRI sequences. For any two IRI pairs,  $(I_m, J_m)$  and  $(I_n, J_n)$ , the model requires that  $\text{cov}(I_m, J_n) = \text{cov}(I_n, J_m)$ . For example, consider the fourth and the sixth IRI-pair of  $\{I_n\}$  and  $\{J_n\}$ , that is,  $(I_4, J_4)$  and  $(I_6, J_6)$ . If the model holds, the amount of covariation between  $I_4$  and  $J_6$  will be the same as that observed when we reserve the role of the hands, that is, between  $J_4$  and  $I_6$ . Of course, the symmetry prediction must hold for all  $m, n > 0$ , that is, for the entire

<sup>e</sup>The estimators used are given by:

$$\widehat{\text{cov}}(I_m, J_n) = \frac{1}{r-1} \left[ \sum_{k=0}^{r-1} I_{m+kc} J_{n+kc} - \frac{1}{r} \sum_{k=0}^{r-1} I_{m+kc} \sum_{k=0}^{r-1} J_{n+kc} \right],$$

where  $r$  = number of cycles per sequence, and  $0 < m, n \leq c$ .

cross-covariance function describing the dependence between the sequences  $\{I_n\}$  and  $\{J_n\}$ . Note that, in general, cross-covariance functions are not symmetric; the predicted symmetry is a consequence of the mutual independence of the three subsystems of the model. Testing for symmetry is thus a way of testing the independence assumption (A2).<sup>f</sup>

The test was carried out by computing the empirical cross-covariance function,  $\text{cov}(I_m, J_n)$ , that relates the IRI sequences  $\{I_m\}$  and  $\{J_m\}$ , making use of the periodicity assumption (A4). For each experimental condition, and  $1 \leq m, n \leq c + 1$ ,  $1 \leq |m - n| \leq c$ ,  $\widehat{\text{cov}}(I_m, J_n)$  was compared with  $\widehat{\text{cov}}(I_n, J_m)$ ;  $c$  is the number of notes per rhythm, that is,  $c = 3$  for condition 2-4-2 and  $c = 4$  for the remaining conditions. FIGURE 5 shows the results combined across conditions. For each subject, each rhythm, and each pair  $(m, n)$ ,  $\widehat{\text{cov}}(I_m, J_n)$  is plotted against  $\widehat{\text{cov}}(I_n, J_m)$ .

If symmetry holds, all points lie on the diagonal except for statistical fluctuation. Evidently, the prediction is borne out rather well since no systematic deviation from the diagonal is discernible.

This conclusion is corroborated by statistical tests; analyses of the sign of  $\widehat{\text{cov}}(I_m, J_n) - \widehat{\text{cov}}(I_n, J_m)$  generally showed deviations in both directions to be about equally likely. Combined across conditions, the  $\chi^2$  statistics (d.f. = 1) obtained for the five subjects were 2.08, .42, 2.95, .86, and .24, respectively, which are all nonsignificant as is the combined statistic,  $\chi^2 = 6.55$  (d.f. = 5). Similarly, combining across subjects, nonsignificant  $\chi^2$  statistics are obtained for the different rhythm conditions (1.69, .07, 1.10, and .30, respectively; d.f. = 1) and their sum (3.16; d.f. = 4). However, when the covariances, collapsed across conditions, are analyzed as a function of lag  $|m - n|$ , some discrepancies are found. For the five subjects, the following  $\chi^2$  statistics (d.f. = 1) resulted: for lag 1: .60, .60, 5.4 ( $p$  .025), 1.67, and 4.57 ( $p$  .05); for lag 2: .08, .08, 1.14, .60, 7.14 ( $p$  .01); and for lag 3: 2.27, .00, .33, .33, 2.78.

The evidence in favor of the symmetry prediction is thus not unequivocal. However, since neither the three discrepant results nor the nonsignificant ones deviate in a consistent direction from the expectations, we tentatively keep the model as a framework within which to examine the stochastic properties of the postulated timing system.

## HIERARCHICAL VERSUS SERIAL TIMING STRUCTURES

### *Unequal Cycle Variances*

If our general model of two-handed tapping is valid, the covariances between left-hand and right-hand IRIs reveal the statistical properties of the central timing system, since  $\text{cov}(I_m, J_n) = \text{cov}(I_n, J_m) = \text{cov}(T_m, T_n)$ . By examining the stochastic structure of  $\{T_n\}$  in detail, we obtain information about the timing mechanisms that underly rhythmic performance. As in Vorberg and Hambuch,<sup>2</sup> our main concern is whether timing is achieved by a *serial* mechanism that simply concatenates the individual intervals within a rhythm, or whether there is evidence for a *hierarchical* organization of the timing system in the sense that the duration of larger units (for example, half-cycles, full cycles) is under direct timer-control rather than controlled indirectly, as in serial timing systems, which time the intervals between successive commands only. In order to detect hierarchically organized timing mechanisms, we

<sup>f</sup>This is an example of the theoretical advantage of describing dependence between the intervals in terms of covariances rather than correlations: in general, symmetry will not hold for the cross-correlation function,  $\rho(I_m, J_n) = \text{cov}(I_m, J_n) / [\text{var}(I_m) \cdot \text{var}(J_n)]^{1/2}$ .

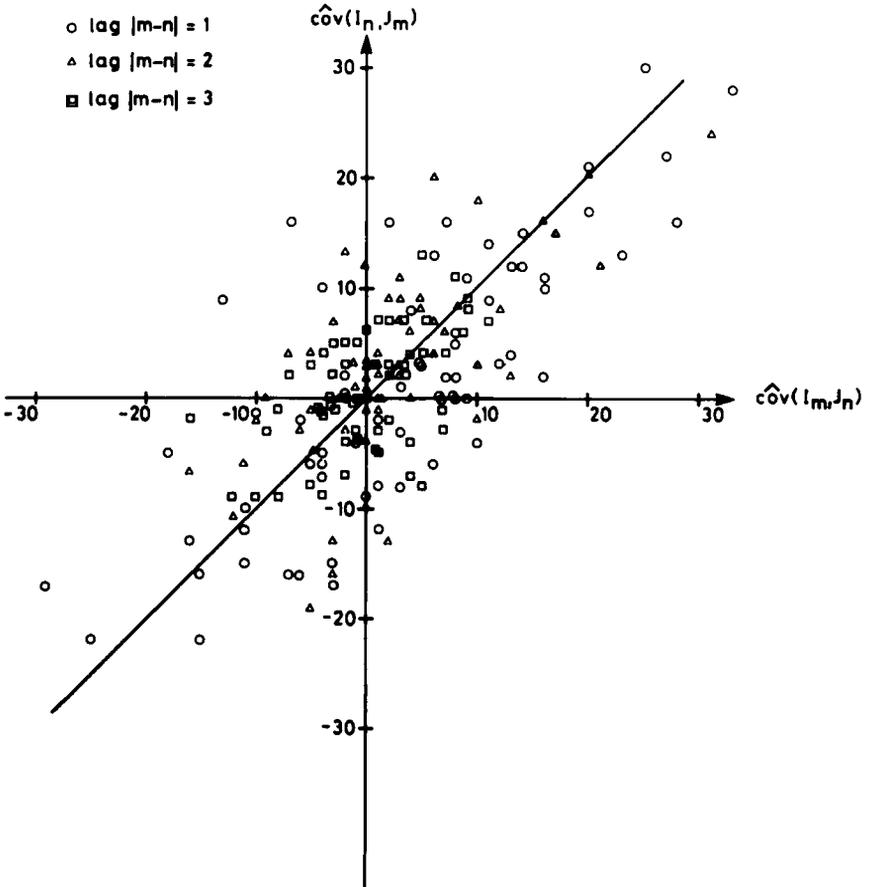


FIGURE 5. Scatter diagram of  $\hat{c}ov(I_n, J_n)$  vs.  $\hat{c}ov(I_m, J_m)$ ,  $0 < m, n \leq c$ ,  $0 < |m - n| < c$ . Each dot represents a different subject times rhythm combination.

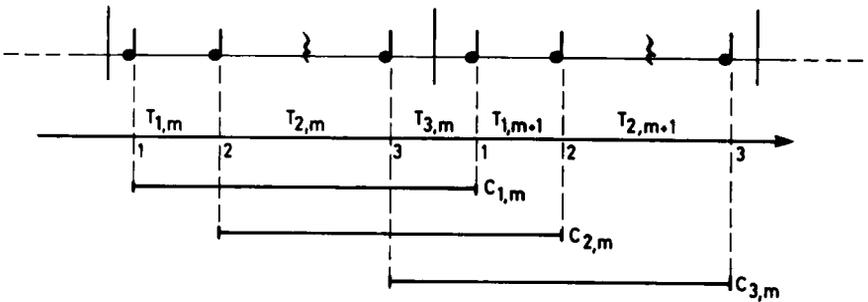


FIGURE 6. Definition of cycle durations,  $C_{i,m}$ , for condition 2-4-2.

examine the precision with which the system controls the duration of cycles. In a cycle consisting of  $c$  notes, there are  $c$  different ways of forming intervals that span full cycles; to determine a cycle's duration, we can either measure the interval bounded by the first notes in successive cycles, or that bounded by the second notes, and so forth. We call these overlapping intervals the alternative *cycle* durations. FIGURE 6 illustrates their definition for rhythm 2-4-2.

For convenience, we change the notation so that it indicates the periodicity of the timer intervals more clearly. Let  $T_{i,m}$  denote the  $i$ th intercommand interval within the  $m$ th cycle, that is, the interval between the  $i$ th command and the following one within the cycle.<sup>8</sup> We then define the cycle durations  $C_{i,m}$ ,  $i = 1, c$ , by

$$C_{i,m} = \begin{cases} \sum_{j=1}^c T_{j,m}, & i = 1 \\ \sum_{j=i}^c T_{j,m} + \sum_{j=1}^{i-1} T_{j,m+1}, & i > 1 \end{cases}$$

(see FIGURE 6).

Since all the  $C_{i,m}$  span one full cycle, they have the same expectation, as is easily seen from *A4*. However, since they differ with respect to the note in the sequence from where they are taken, their variances need not be equal. In fact, if there exist mechanisms that directly time the duration of intervals larger than those between successive commands, the variance of cycle durations that contain those higher-order timing units will in general be smaller than of those that start and end in different higher-order units.

These intuitive ideas can be made more precise. Let us analyze the variances of the cycle durations, called *cycle variances* in the following, by going back to the definition of the  $C_{i,m}$ . As an example, consider cycles with  $c = 3$ . For  $C_{1,m}$  and  $C_{2,m}$ , we obtain

$$\begin{aligned} \text{var}(C_{1,m}) &= \text{var}(T_{1,m} + T_{2,m} + T_{3,m}) \\ &= \text{var}(T_{1,m} + (T_{2,m} + T_{3,m})) \\ &= \text{var}(T_{1,m}) + \text{var}(T_{2,m} + T_{3,m}) + 2 \text{cov}[T_{1,m}, (T_{2,m} + T_{3,m})]; \\ \text{var}(C_{2,m}) &= \text{var}(T_{2,m} + T_{3,m} + T_{1,m+1}) \\ &= \text{var}[(T_{2,m} + T_{3,m}) + T_{1,m+1}] \\ &= \text{var}(T_{2,m} + T_{3,m}) + \text{var}(T_{1,m+1}) + 2 \text{cov}[(T_{2,m} + T_{3,m}), T_{1,m+1}]. \end{aligned}$$

Since  $\text{var}(T_{1,m+1}) = \text{var}(T_{1,m})$  due to the periodicity of the timer intervals, we see that  $\text{var}(C_{1,m}) \leq \text{var}(C_{2,m})$  if and only if  $\text{cov}[T_{1,m}, (T_{2,m} + T_{3,m})] \leq \text{cov}[(T_{2,m} + T_{3,m}), T_{1,m+1}]$ . In a similar way we obtain analogous inequalities for all pairs of cycle variances. The results are summarized by

$$\text{var}(C_{1,m}) \leq \text{var}(C_{2,m}) \tag{3a}$$

if and only if

$$\begin{aligned} \text{cov}[T_{1,m}, (T_{2,m} + T_{3,m})] &\leq \text{cov}[(T_{2,m} + T_{3,m}), T_{1,m+1}], \\ \text{var}(C_{1,m}) &\leq \text{var}(C_{3,m}) \end{aligned} \tag{3b}$$

<sup>8</sup>The different notations are related by  $T_n = T_{i,m}$  if  $n = c(m - 1) + i$ .

if and only if

$$\begin{aligned} \text{cov} [(T_{1,m} + T_{2,m}), T_{3,m}] &\leq \text{cov} [T_{3,m}, (T_{1,m+1} + T_{2,m+1})], \\ \text{var} (C_{2,m}) &\leq \text{var} (C_{3,m}) \end{aligned}$$

if and only if

$$\text{cov} [T_{2,m}, (T_{3,m} + T_{1,m+1})] \leq \text{cov} [(T_{3,m} + T_{1,m+1}), T_{2,m+1}]. \quad (3c)$$

To see the implications of these inequalities, consider Equation 3a. If rhythmic performance is timed by a serial mechanism that directly controls the duration of the lowest-order intervals only, there is no reason why the sum of the second and third intervals of a cycle should covary more with the first interval of the same cycle than with that of the following one. If, on the other hand, there is a higher-order mechanism within a hierarchical timing system controlling the duration of the whole cycle,<sup>h</sup> that is,  $C_{1,m} = T_{1,m} + T_{2,m} + T_{3,m}$ , we should expect a lower covariance between the segments within a cycle, that is,  $T_{1,m}$  and  $T_{2,m} + T_{3,m}$ , than across two cycles, that is,  $T_{2,m} + T_{3,m}$  and  $T_{1,m+1}$ . This is because time intervals from different cycles will in general be either independent or covary slightly positively with each other due to tempo fluctuations, whereas intervals within cycles ought to covary less; if the first interval,  $T_{1,m}$ , happens to be smaller (larger) than average, the sum of the following two intervals,  $T_{2,m} + T_{3,m}$ , will be likely to be larger (smaller) than average by virtue of the higher-order timer controlling the total cycle duration,  $C_{1,m}$ . Similar reasoning can be applied to the remaining inequalities as well, leading to the conclusion that the variances of  $C_{1,m}$ ,  $C_{2,m}$  and  $C_{3,m}$  should be about the same if the underlying timer system is serial, whereas systematic differences in the cycle variances are to be expected if rhythmic performance is controlled by a hierarchical timing system with higher-order units that time the duration of intervals between nonsuccessive commands. Of course, the analogous prediction holds for the cycle variances of rhythms with any number of notes within a cycle,  $c$ .

### Particular Hierarchical Models

We can sharpen our notion of hierarchical timing structures and gain more insight into their behavior by modeling them. Previously, we<sup>2</sup> have considered a class of timing models defined by the following properties: (i) A timing structure controlling a rhythmic cycle with  $c$  notes consists of  $c$  timers that may be located on different levels of a hierarchy. (ii) When started, a timer generates an interval; when it expires, the timer may start other timers *on the same or on lower levels* of the hierarchy. (iii) A timer is started by exactly one other timer (which may be itself if it is on the top level). (iv) Low-level timers generate intervals that are overlapped completely by those generated on higher levels.

As an example, we examine in some detail the two-level timing structure shown in FIGURE 7. This level-2 timer controls the duration of the full cycle. The successive intervals it generates, indicated by the arrows, are random variables denoted by  $F_m$ ,  $F_{m+1}$ , . . . . The timer restarts itself when the last time interval has elapsed. Simultaneously, a level-1 timer is started that controls the interval between the first

<sup>h</sup>For simplicity, we are assuming here that the top-level timer starts on the cycle's first note. The argument remains valid if timing starts from a note different from the nominal beginning of a cycle.

two notes in the cycles, i.e.,  $T_{1,m}$ . When it expires, a second level-1 timer is triggered which determines the interval between the second and the third note, that is,  $T_{2,m}$ . The intervals generated by these level-1 timers are denoted by  $D_m$  and  $E_m$ , respectively.

The gist of the notion of a hierarchical timing structure is that timing control may pass from higher to lower levels, but not vice versa. Therefore, the interval between the last note in the  $m$ th cycle and the first one of the  $(m + 1)$ th cycle,  $T_{3,m}$ , is not controlled directly, but is determined by the joint operation of the level-2 and the level-1 timers. FIGURE 7 shows that  $T_{3,m} = F_m - D_m - E_m$ .

Hierarchical timing structures may be described in terms of the relations between the time intervals bounded by successive commands, the  $T_{i,m}$ , and the intervals generated by the separate timing devices. For the particular structure, these are given by

$$T_{1,m} = D_m, \tag{4a}$$

$$T_{2,m} = E_m, \tag{4b}$$

$$T_{3,m} = F_m - D_m - E_m. \tag{4c}$$

In principle, the  $\{D_m\}$ ,  $\{E_m\}$  and  $\{F_m\}$  may be arbitrary sequences of non-negative

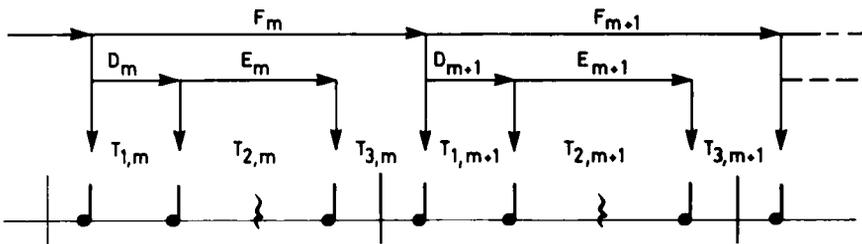


FIGURE 7. A hierarchical timing structure for cycles with  $c = 3$ .

random variables, except for the constraint that  $P(F_m \geq D_m + E_m) = P(T_{3,m} \geq 0) = 1$ , since otherwise the proper order of the notes within cycles would not necessarily be maintained. Of course, this is the analogue of the order preservation assumption (A3) that is to hold for the joint operation of the timer and the motor systems.

Consider the ordering of the cycle variances predicted by this particular model. For example, from Equation 4a, inequality 3a can be rewritten as

$$\text{var}(C_{1,m}) \leq \text{var}(C_{2,m}) \leftrightarrow \text{cov}(D_m, F_m - D_m) \leq \text{cov}(F_m - D_m, D_{m+1}).$$

In Vorberg and Hambuch,<sup>2</sup> we assumed all the timer random variables to be mutually independent. This leads to the expected variance ordering, that is,  $\text{var}(C_{1,m}) \leq \text{var}(C_{2,m})$ , since  $\text{cov}(D_m, F_m - D_m) = -\text{var}(D_m)$ , and  $\text{cov}(F_m - D_m, D_{m+1}) = 0$  for independent  $D_m$ ,  $D_{m+1}$  and  $F_m$ . It is important to note, however, that our technique of inferring the existence of hierarchical timing mechanisms from unequal cycle variances is more general and may remain valid even if independence breaks down.

For example, assume that independence only holds between intervals generated by different timers, but that successive intervals produced by the same timer covary. This still implies  $\text{var}(C_{1,m}) \leq \text{var}(C_{2,m})$  since  $\text{cov}(D_m, F_m - D_m) = -\text{var}(D_m) \leq -\text{cov}(D_m, D_{m+1}) = \text{cov}(F_m - D_m, D_{m+1})$ , given that  $\text{cov}(D_m, F_m) = \text{cov}(D_{m+1}, F_m) = 0$ .

Another, more reasonable, assumption might be that the intervals produced within any given cycle are dependent, which would be the case if the hierarchy passed fallible information down its levels for setting the timer parameters. This is likely to cause positive dependence between the intervals within cycles, that is,  $\text{cov}(D_m, F_m) \geq 0$  and  $\text{cov}(D_m, E_m) \geq 0$  in our example, since the information used to set the level-2 timer is also fed to the level-1 timers. If independence is assumed between intervals within different cycles only, the variance of the cycle duration taken from the second note,  $\text{var}(C_{2,m})$ , will, under quite general conditions, still exceed that of the "natural" cycle duration,  $\text{var}(C_{1,m})$ . If  $\text{cov}(F_m, D_{m+1}) = \text{cov}(D_m, D_{m+1}) = 0$ , the result is  $\text{var}(C_{1,m}) \leq \text{var}(C_{2,m}) \leftrightarrow \text{cov}(D_m, F_m) \leq \text{var}(D_m)$ . By some algebra, this can be shown to be equivalent to the condition  $\rho_{DF} \leq \sigma_D/\sigma_F$ . Thus, for negative as well as for some amount of positive dependence between the timer-produced intervals, the predicted variance ordering holds.

Additional inequalities relating the variances of the alternative cycle durations,  $C_{1,m}$ ,  $C_{2,m}$ , and  $C_{3,m}$ , can be derived from (3) and (4) for this particular model, and, of course, analogously for models that embody alternative timing structures as well as for arbitrary numbers of notes per cycle.

There exist 10 different distinguishable models for rhythms with  $c = 3$ , and 31 for rhythms with  $c = 4$  satisfying conditions (i) to (iv) above. Rather than investigating them all, we make use of the cycle variance inequalities as a diagnostic means for determining some kind of hierarchical timing structure and as a test against serially organized timing mechanisms. For the whole class of models described above it can be shown quite generally that some unequal cycle variances are to be expected unless the timing structure is serial; this prediction holds for a wide range of dependence between the different timer random variables.

**Results**

In order to estimate the cycle variances,  $\text{var}(C_{i,m})$ , covariances were computed for appropriately concatenated left-hand and right-hand IRIs; for example, for  $c = 3$  the estimate of  $\text{var}(C_{1,m})$  is given by  $\widehat{\text{cov}}(I_{1,m} + I_{2,m} + I_{3,m}, J_{1,m} + J_{2,m} + J_{3,m})$ , since  $C_{1,m} = T_{1,m} + T_{2,m} + T_{3,m}$ .

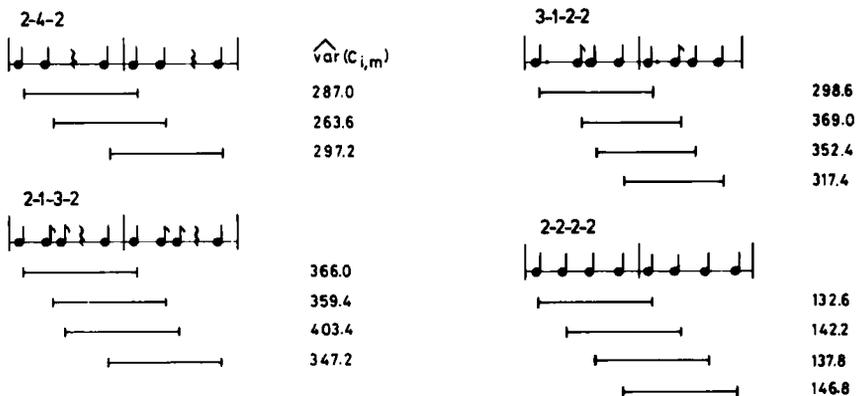


FIGURE 8. Average cycle variances obtained for the different rhythms.

TABLE 1. Central Timer Autocovariances Estimated from the Cross-Covariances Between Left-Hand and Right-Hand IRI Sequences

Condition 2-4-2						
	$T_{2,m}$	$T_{3,m}$	$T_{1,m+1}$	$T_{2,m+1}$		
$T_{1,m}$	3.5	5.5**				
$T_{2,m}$		-8.5**	2.5			
$T_{3,m}$			-5.5**	8.0**		
Condition 2-1-3-2						
	$T_{2,m}$	$T_{3,m}$	$T_{4,m}$	$T_{1,m+1}$	$T_{2,m+1}$	$T_{3,m+1}$
$T_{1,m}$	4.0*	2.0	5.0**			
$T_{2,m}$		-11.5***	-1.0	1.5		
$T_{3,m}$			15.0***	1.0	-2.0	
$T_{4,m}$				5.0	9.5***	-4.5
Condition 3-1-2-2						
	$T_{2,m}$	$T_{3,m}$	$T_{4,m}$	$T_{1,m+1}$	$T_{2,m+1}$	$T_{3,m+1}$
$T_{1,m}$	-4.5	-7.5*	4.0			
$T_{2,m}$		2.5	4.0**	.5		
$T_{3,m}$			1.5	11.0**	.0	
$T_{4,m}$				16.0***	2.5	-1.0
Condition 2-2-2-2						
	$T_{2,m}$	$T_{3,m}$	$T_{4,m}$	$T_{1,m+1}$	$T_{2,m+1}$	$T_{3,m+1}$
$T_{1,m}$	-.5	2.5	-2.5			
$T_{2,m}$		1.5	3.5*	-.5		
$T_{3,m}$			3.5***	-1.5	.0	
$T_{4,m}$				6.0***	4.0*	.5

NOTE: Asterisks indicate covariances that differ significantly from zero (\*:  $p < .05$ ; \*\*:  $p < .01$ ; \*\*\*:  $p < .001$ ).

The estimates obtained for the different cycle variances, averaged over subjects, are shown in FIGURE 8. Analyses of variance (repeated measures design), performed on the square roots of these estimates, showed the differences to be significant except under the isochronous condition (2-4-2:  $F(2,8) = 96.38, p < .001$ ; 2-1-3-2:  $F(3,12) = 3.80, p < .05$ ; 3-1-2-2:  $F(3,12) = 14.06, p < .001$ ; 2-2-2-2:  $F(3,12) = 1.47, p > .25$ ). Thus, there is reliable evidence for the existence of higher-order timing mechanisms controlling the production of rhythms with unequal time intervals, whereas isochronous rhythms seem to be controlled by a simple serial timing structure. This last finding is consistent with those reported by Vorberg and Hambuch.<sup>2</sup> Analyzing response sequences from experimental conditions where subjects were required to tap evenly with one hand while grouping their responses by two, three, or four, we found no evidence supporting hierarchical timing structures, whereas the data were in agreement with the predictions of a serial timing structure.

The results of the cycle variance test rule out serial timing mechanisms for conditions 2-4-2, 2-1-3-2, and 3-1-2-2; however, they do not tell us much about the particular structure of the inferred timing mechanisms. Unfortunately, none of the strong hierarchical models that we have proposed<sup>2</sup> with timing structures composed of independent random variables is consistent with the data, as a cursory examination of the timer covariances reveals. TABLE 1 shows the  $\hat{cov}(T_{i,m}, T_{j,m+k})$ , estimated from the averaged cross-covariances,  $[cov(I_{i,m}, J_{j,m+k}) + cov(J_{j,m+k}, I_{i,m})]/2$ . For each condition, the estimated timer covariances are given for intervals no more than  $c - 1$  steps apart.

Hierarchical models with independent timer random variables have problems with

these data since they cannot account for positive covariances between any pair of intervals,  $T_n = T_{i,m}$  and  $T_q = T_{j,m+k}$ , unless  $n = q$ . It is easy to show that, in general,  $\text{cov}(T_n, T_q) \leq 0$ ,  $n \neq q$ , for all models of the class sketched above. However, as TABLE 1 shows, there are sizable positive covariances under each condition in addition to the negative ones that are expected for hierarchical models. The reliability of these findings was assessed by testing the average estimates against the corresponding standard errors. The results clearly show timing structures that are more complex than our present models can account for.

## DISCUSSION

The proposed general framework for two-handed tapping has performed rather well in the test of the symmetry prediction as well as in several others reported in Vorberg and Hambuch.<sup>7</sup> We take this as support for the assumptions of a common timing system and of independence between the three subsystems. This conclusion is in agreement with several recent findings bearing on the assumptions.

Wing<sup>6</sup> has reported the application of a related model to two-handed even tapping; his model admits positive covariation between the left-hand and the right-hand motor delays by positing that they share a common component,  $C'_m$ , such that  $L_m = C'_m + L'_m$ , and  $R_m = C'_m + R'_m$ , where  $C'_m$ ,  $L'_m$ , and  $R'_m$  are assumed to be independent. The model implies that  $\text{cov}(L_m, R_m) = \text{var}(C'_m)$ . However, Wing's analysis showed that the contribution of the postulated common component was negligible, that is,  $\text{var}(C'_m) \approx 0$ , implying independence between the left-hand and the right-hand motor delays.

Shaffer<sup>3</sup> has analyzed actual piano performance in terms of a model that is identical to ours except that the random variables are assumed to be independent within systems as well. Several of the model's predictions could not be supported; however, Shaffer concluded that the model fits the data well if the motor delays within hands are allowed to covary. Both this as well as Wing's result can be regarded as additional support for the weaker general model which requires independence between systems only.

The interpretation is valid if the two hands are controlled by the same timing system for which we do not have direct evidence. However, in view of the findings of Kelso *et al.*<sup>4</sup> that there is a strong tendency for the two hands to start and stop movements in synchrony even when they have to perform different movements, the common timing system assumption seems reasonable under the conditions of our experiments. This does not mean that separate timing systems for the two hands might not be involved in more complex rhythmic performance as in polymeric and polyrhythmic music, or in rubato passages in piano playing where one hand moves systematically in and out of the meter provided by the other hand.<sup>3,11</sup> The large amounts of practice most musicians require to achieve such levels of proficiency, however, can be taken as evidence for a single timing system under conditions where this is sufficient.

The model provides an easy way to assess the properties of the timing system directly via the cross-covariances of the IRIs between hands. This is the major advantage of the approach we advocate. In contrast to previous work on one-handed performance, our conclusions about the timing system's structure do not depend on particular assumptions about the motor delays (for example, equal variances, independence within hands). Nevertheless, our findings agree well with our earlier work<sup>2</sup>; both studies found no evidence in support of hierarchical structures for isochronous rhythms.

Earlier,<sup>2</sup> we speculated about the advantages of serial as compared to hierarchical timing structures. We pointed out that hierarchical timing necessitates control mechanisms that prevent the order of responses set up by the motor program from scrambling during its execution, whereas serial structures, directly timing the intervals between successive commands, do not face this problem. A related point is that hierarchical structures with concurrent timers will have to control longer intervals than a serial structure appropriate for the rhythm at hand. This has implications for the temporal precision with which a serial structure can control the duration of a cycle as compared to that of hierarchical ones: We might expect serial timing to be able to keep the variance of any of the cycle durations smaller than hierarchically organized timers can. This expectation is borne out in the data. The cycle variances under the isochronous condition are only about half as large as those of the remaining conditions which were concluded to be controlled hierarchically. Thus, the qualitative differences that we observed between isochronous rhythms as compared to rhythms composed of unequal intervals are paralleled by a quantitative difference in timing precision, corroborating our conclusion that rhythmic performance seems to be timed by some kind of hierarchical structure unless its notes are evenly spaced in time.

The most interesting question remains: What are the particular structures that underlie performance under conditions 2-4-2, 2-1-3-2, and 3-1-2-2? The covariance estimates (TABLE 1) seem to provide some clues. However, we must admit that we have been unable so far to extract meaningful patterns from these data. What is needed are models of timing structures that can account for positive dependencies in the timer intervals as well as for negative ones. At present, we are working on models that generalize the class of hierarchical models sketched above. We assume that each timer in a hierarchy uses information about the size of the interval to be generated which is provided by the timer above it in the hierarchy, and passes it along to those timers below that it dominates. In this way, positive dependence is introduced between the different timer random variables; its extent depends on the variability of the information used to set the timer parameters. Whether such models can give a satisfactory account of our findings remains to be seen. Before fitting them to data, it seems necessary to establish the generality of our present results by extending our analyses to situations which involve different tempos as well as more complex movement sequences. Research along these lines is currently under way.

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