Minimal Energy Control of a Biped Robot with Numerical Methods and a Recursive Symbolic Dynamic Model

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Abstract

The problem of constructing a nonlinear controller for a biped robot optimal with respect to a minimal energy performance criteria is considered. The solution of this difficult, highly nonlinear problem is facilitated by the conjunction of several new developments in numerical optimal control and constrained recursive dynamic models for robotic systems. A 5-link biped model is used with the full dynamics and a uniform distribution of mass at each link. Contacts are modeled as inelastic, and the full dynamics together with the contact and collision forces are calculated efficiently using a recursive symbolic representation of the dynamics. The flexibility and modularity of our dynamics algorithms allows one to construct reduced unconstrained models which do not suffer from integration difficulties. The numerical optimal control software used is powerful and quick enough to handle high dimensional nonlinear systems. The result of our experiment is a walking controller which is optimal with respect to a type of minimum energy performance.

1 Introduction

The problem of controlling the walking motion of a biped robot has intrigued and challenged researchers for many years [2, 4, 6, 7, 8, 10]. This problem presents many obstacles which often can be difficult to avoid without making very compromising simplifying modeling assumptions. Even a simple 5-link biped robot with all rotational joints and full motion degrees of freedom will have a 16 dimensional state space when represented with respect to generalized coordinates. The high dimensionality is additionally compounded with impulsive and one-sided contact forces as a result of collision and contact with the environment.

In order to handle the complexity of the problem, some researchers have made large simplifying assumptions such as modeling the biped as an inverted pendulum [7]. Others have used accurate dynamical models, yet joint trajectories were estimated and linear feedback was used to stabilize them [10]. Our experiments have shown that many walking trajectories naively chosen to approximate walking motion can require a huge increase in the needed power over that of the optimally calculated minimum energy trajectory. For this reason, we take the approach found in the work of McGeer [8] and later with Goswami et al. [4] in that we seek to generate natural walking. The minimal energy path is desirable for it exhibits stabilizing, attractive properties.

During approximately 10%-15% of the human walking step, both feet are in contact with the ground and a closed-chain system exists [2]. Closed-chain systems can be difficult to handle as they are high order differential-algebraic systems, yet the influence of ground collisions and double contact forces is important, and we choose not to neglect them.

In this paper, we will briefly present our reduced dynamics algorithm. Using a symbolic representation of the dynamics, we efficiently calculate the full contact dynamics which we then use in conjunction with the numerical optimal control package DIRCOL [11, 12]. The reduced dynamics algorithm allows DIRCOL to work with a model of reduced dimension where the contact constraints have been exploited. The result is an optimal open-loop control law which generates a periodic walking motion.

2 Model and Dynamics

In numerical nonlinear control, one normally requires the generalized accelerations $\ddot{\theta}$ which are the time derivative of the state. This involves evaluating the forward dynamics equation

$$\ddot{\theta} = \mathcal{M}^{-1}(u + J_c^T f_c - \mathcal{C} - \mathcal{G}) \ . \qquad (1)$$

In equation (1), $\mathcal{M}$ is the square, positive-definite mass-inertia matrix, $\mathcal{C}$ is the vector of Coriolis and centrifugal forces, $\mathcal{G}$ is a vector of gravitational forces, u are the applied torques at the links, J_c is the constraint Jacobian, and f_c is the constraint force.

A 5-link biped robot with motion only allowed in the sagittal plane, the plane of forward motion which bi-

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sects the robot, has already 7 degrees of freedom. Already in this case, recursive symbolic dynamical models are more efficient for calculating the forward dynamics than other procedures which require constructing and inverting the entire mass-inertia matrix $\mathcal{M}$. $\mathcal{O}(\mathcal{N})$ recursive algorithms, where $\mathcal{N}$ is the number of degrees of freedom of the system, are advantageous not only for increased efficiency with more degrees of freedom. Due to the symbolic nature of the algorithm, changes to the kinematic or dynamical parameters are easily made.

The modeling approach used here is that found in [9, 1, 5]. Most of the recursive symbolic algorithms found in the literature use the same principles in their algorithms. The one described in [9] and [1], and finally in [5] is already a mature one since many aspects of robot dynamics and control have been studied and developed including contact and collision dynamics, tree structured systems, and multiple degree of freedom joints.

We give now a brief summary of the **Contact** and **Collision Algorithms**. If the tip contact constraint (free foot touching the ground) is given holonomically as $c(\theta) = 0$, then by taking time derivatives we also obtain

$$\begin{aligned} J_c\dot{\theta} &= 0 & (2)\\ J_c\ddot{\theta} + \dot{J}_c\dot{\theta} &= 0\ . & (3) \end{aligned}$$

where $J_c = \partial c/\partial\theta$. Multiplying (1) by J_c and substituting for $J_c\ddot{\theta}$ using (3), one obtains an operator expression for f_c.

$$\begin{aligned} f_c &= (J_c\mathcal{M}^{-1}J_c^T)^{-1}[-J_c\mathcal{M}^{-1}(u - \mathcal{C} - \mathcal{G}) - \dot{J}_c\dot{\theta}]\\ &= -\Omega^{-1}(J_c\ddot{\theta}_f + \dot{J}_c\dot{\theta})\ ,\\ &= -\Omega^{-1}Q\dot{V}\ . \qquad (4) \end{aligned}$$

$\Omega^{-1} = (J_c\mathcal{M}^{-1}J_c^T)^{-1}$ is a square matrix of dimension equal to the number of constraints, and it is a quantity related to the Khatib operational space inertia. $\ddot{\theta}_f$ are the free generalized accelerations without the influence of the contact force in the dynamics while the final expression for f_c includes the spatial acceleration $\dot{V}$, the time derivative of V. The contact constraints of a branch with the ground can be expressed in terms of the spatial acceleration of the branch tip $\dot{V}_{N(i)+1}$, which contains its angular and linear acceleration in inertial coordinates. The acceleration constraints (3) may then be written together with a constant matrix Q as $Q\dot{V} = d/dt(J_c\dot{\theta}) = 0$.

The true angle accelerations are the sum of $\ddot{\theta}_f$ and a correction term $\ddot{\theta}_\delta$ which results from the contact forces propagating throughout the body. These correction accelerations can be calculated from f_c by the relationship

$$\ddot{\theta}_\delta = \mathcal{M}^{-1}J_c^T f_c\ . \qquad (5)$$

A very similar algorithm exists for calculating the change in velocities due to an inelastic collision with the ground. The change in the generalized velocities will depend on the tip velocities at the moment of contact with the ground. One solves for the impulse force f_{imp},

$$f_{imp} = -\Omega^{-1}QV\ . \qquad (6)$$

One may solve for $\dot{\theta}_\delta$ in $\dot{\theta}_\delta = \mathcal{M}^{-1}J_c^T f_{imp}$ to obtain the generalized velocities after collision $\dot{\theta}_+ = \dot{\theta}_- + \dot{\theta}_\delta$. The **Contact** and **Collision Algorithms** are discussed at greater length in [1].

3 Reduced Dynamics Algorithm

With the introduction of homonomic constraints, such as the contact of feet with the ground, it is possible to construct a set of reduced unconstrained dynamics of dimension equal to the number of degrees of freedom minus the number of constraints. In this section, we outline our approach to calculating the independent generalized accelerations of the reduced set of dynamics. The novelty of this approach is that it does not require the explicit construction of the reduced dynamics. It will be shown how one may extract the solution of the reduced dynamics from the solution of the contact algorithm and the solution of the forward dynamics of the entire system. One main advantage of using a reduced unconstrained dynamical model is that optimization programs which require integration of the dynamics will encounter less numerical difficulties.

In order to satisfy the constraint condition (2), the generalized velocities $\dot{\theta}$ must belong to the null space of the constraint Jacobian, $\mathcal{N}(J_c) \subset R^{N-m}$. If the columns of X represents a basis for $\mathcal{N}(J_c)$, then there exists a representation of $\dot{\theta}$ with respect to X denoted here by ξ, $\dot{\theta} = X\xi$. Substituting $\ddot{\theta} = X\dot{\xi} + \dot{X}\xi$ into the dynamical equations and multiplying on the left by X^T will give us the reduced dynamics,

$$\mathcal{M}_\xi\dot{\xi} + \mathcal{C}_\xi + \mathcal{G}_\xi = u_\xi\ , \qquad (7)$$

where $\mathcal{M}_\xi = X^T\mathcal{M}X$, $\mathcal{C}_\xi = X^T\mathcal{M}\dot{X}\xi + X^T\mathcal{C}$, $\mathcal{G}_\xi = X^T\mathcal{G}$, and $u_\xi = X^Tu$.

If θ represents the generalized coordinates of the system, then it is possible to choose $N - m$ independent coordinates θ_1 and m dependent coordinates θ_2 such that $J_{c,1}\dot{\theta}_1 + J_{c,2}\dot{\theta}_2 = 0$ may be used as an alternative expression for (2). This approach was made in [10], and it leads to an obvious choice for X,

$$\dot{\theta} = X\xi = X_0\dot{\theta}_1 = \begin{bmatrix} I \\ -J_{c,2}^{-1}J_{c,1} \end{bmatrix}\dot{\theta}_1\ . \qquad (8)$$

An advantage of making this choice for the basis X is that, as will be shown in the **Reduced Dynamics Algorithm**, the reduced accelerations are simply represented as $\dot{\xi} = \ddot{\theta}_1$. Our goal here is to show that the solution of the contact algorithm may be used to obtain a solution of the reduced forward dynamics problem. Then an optimization routine performing numerical integration need only integrate on the independent coordinates $\ddot{\theta}_1$. We first give a lemma before the algorithm is presented.

Lemma 1 *Let X be a basis for the null space of the constraint Jacobian, $\mathcal{N}(J_c)$, and assume that at time $t=0$, the state $(\theta, \dot{\theta})$ satisfies the constraint conditions $c(\theta(0)) = 0$, $J_c\dot{\theta}(0) = 0$. Then the following statements are equivalent:*

(a) $\dot{\theta}$ and $\ddot{\theta}$ satisfy $J_c\ddot{\theta} + \dot{J}_c\dot{\theta} = 0$.

(b) There exists an $N-m$ dimensional vector ξ which satisfies $\dot{\theta} = X\xi$.

(c) $X\dot{\xi} = \ddot{\theta} - \dot{X}\xi$ is consistent and has a unique solution $\dot{\xi}$.

Proof: $(c) \Rightarrow (a)$ Since X is a basis for $\mathcal{N}(J_c)$, then $J_cX\xi = 0$ and $d/dt(J_cX\xi) = 0$. So,

$$J_c\ddot{\theta} + \dot{J}_c\dot{\theta} = J_cX\dot{\xi} + (J_c\dot{X} + \dot{J}_cX)\xi = 0 \ .$$

$(a) \Rightarrow (b)$ Integrating (a) implies $J_c\dot{\theta} = 0$ since $J_c\dot{\theta}(0) = 0$ at time $t=0$. $J_c\dot{\theta} = 0$ further implies that there exists a representation ξ for $\dot{\theta}$ with respect to X, $\dot{\theta} = X\xi$. $(b) \Rightarrow (c)$ Differentiating $\dot{\theta} = X\xi$ and observing that X is full rank gives the desired result. ■

Reduced Dynamics Algorithm

1. Beginning with an independent set of coordinates $\xi = \theta_1$, solve via inverse kinematics for θ_2 from θ_1. Similarly solve for $\dot{\theta}_2$ from $\dot{\theta}_1$ using the algebraic relation $\dot{\theta}_2 = -J_{c,2}^{-1}J_{c,1}\dot{\theta}_1$.
2. Given a set of torque inputs u, one may solve for $\ddot{\theta}$ with the contact algorithm. Simple algebraic manipulation shows that this solution satisfies (a) of Lemma 1.
3. Using Eq. (8), it follows that $\dot{\xi} = \ddot{\theta}_1$, and it satisfies the reduced dynamics (7). .

This algorithm thus yields the reduced forward dynamics mapping $u \rightarrow (\xi, \dot{\xi})$.

4 Numerical Optimal Control

Direct optimization methods for optimal control are characterized by the minimization of a cost functional which is a function of the system state and the control input u. An example of such a method is the program DIRCOL [11, 12], which can handle implicit or explicit boundary conditions, arbitrary nonlinear equality and inequality constraints on the state variables, and multiple phases where each phase may contain a different set of state equations. DIRCOL functions by packaging the optimal control problem along with its constraints into a constrained nonlinear minimization problem which is solved by an SQP-based optimization code NPSOL (Gill, Murray, Saunders, Wright [3]).

Our performance objective in this problem is a criteria related to minimum energy or minimum power provided by the controller. We minimize the performance equal to the integral of the squared magnitudes of the torque inputs,

$$\min_u \int_0^T ||u(t)||^2 \, dt \ . \tag{9}$$

DIRCOL discretizes the state and control variables in time over the trajectory. The output of the numerical optimal control program will be the optimal open-loop solution for the control $u(t)$ and the corresponding state trajectory $x(t)$ at the choice of grid points in time. With regards to the biped walking control problem, the final values of $u(t)$ and $x(t)$ at the end of a half step will be symmetrically constrained to match the initial values for u and x at the beginning of the step. This produces an optimal periodic solution for the desired walking trajectory.

After the optimal periodic trajectory is calculated, one can solve the problem for initial states off the periodic trajectory. This procedure can then be repeated throughout a grid of initial states in a region surrounding the periodic trajectory such that a closed-loop control law can be closely approximated given any deviations from the walking motion within this region.

5 Experiment

Our experiment was made with a 5-link biped robot. The most restrictive assumption made is that we consider only motion in the sagittal plane, the plane of forward motion which bisects the robot. The links are modeled with an elliptical shape and a uniform distribution of mass.

The physical model of the biped walker used here, as mentioned previously, contains five links. The physical data corresponding to the model are found in Table 1. The experiment is conducted at a desired walking speed of $0.5 m/s$.

Table 1: Model Physical Data

Link	Mass	Length	Radius
Torso	20 kg	0.72 m	0.12 m
Upper Leg	7 kg	0.50 m	0.07 m
Lower Leg	4 kg	0.50 m	0.05 m

The optimization in DIRCOL consisted of two phases. The first phase was the swing phase of the leg and was modeled by an unconstrained single-chain manipulator. The second phase, which accounted for 15% of the total step time, was characterized by having both feet on the ground with the torso continuing to move forward until the time when the back leg was ready to leave the ground. In this phase, we made use of the **Reduced Dynamics Algorithm**. Finally, the boundary conditions in between phases required the evaluation of the

Collision Algorithm which produced a discontinuous jump in the generalized velocities and a corresponding loss of kinetic energy as a result of the collision of the foot with the ground. The only walking parameters passed to DIRCOL were the length of stride, the duration of the half step, and the percentage of time in the double support phase.

In our opinion, the experiment produced a very 'natural' walking motion of which some frames are shown in Figure 1. If one takes the control inputs u necessary to generate a particular walking step, then one can evaluate the cost of the step using the performance (9). Remarkable was the extremely large drop in the cost of making the optimal step calculated here compared to other methods which attempt to estimate the best trajectory purely from visual information.

6 Future Directions

Our recursive dynamics model gives one the flexibility to choose which link in the robot is to serve as the base. For the experiment described in the last section, the foot which is at all times in contact with the ground during a half step was chosen as the base. This choice made it possible to model the biped as a single-chain manipulator. Presently, we are working on expanding the modeling framework to allow for general tree-structured systems. This approach will be necessary when other motions and contact situations are investigated such as running.

For the construction of the feedback controller, there are many interesting new alternatives available. In particular, new evolutionary programming tools are being investigated for constructing closed form algebraic expressions which approximate both the periodic optimal walking trajectory and the optimal closed-loop control inputs.

7 Conclusion

We solve numerically the problem of minimal energy control of a biped walker. The overall method is efficient with the use of recursive dynamic algorithms which easily allow changes to kinematic and dynamic configurations. One of the main contributions of this paper is a new algorithm for recursively calculating the reduced set of dynamics corresponding to contact constraints with the environment. Previous results for recursive symbolic dynamic modeling techniques and recursive constraint projection algorithms are joined in the Reduced Dynamics Algorithm. The synthesis of these modeling tools with recently developed numerical optimal control software allows us to tackle the challenging problem analyzing and optimizing the contact equations of motion. The software efficiently calculates optimal open-loop controls which generate trajectories representing a 'natural' walking motion.

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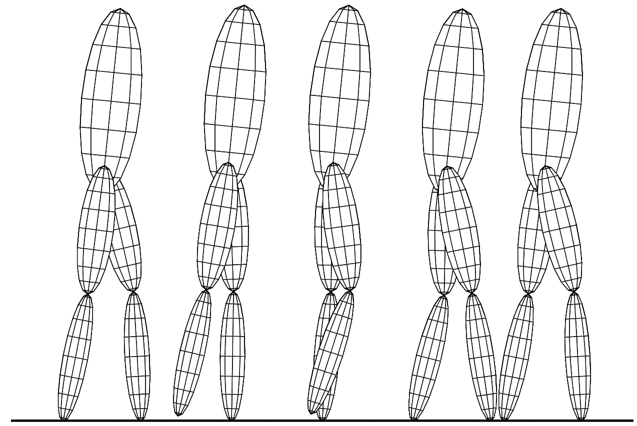

Figure 1: Walking Motion

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